\[ F = Uz + \frac{M}{2\pi} \ln \left( \frac{z+1}{z-1} \right) \]

\[ = Uz + \frac{M}{2\pi} \ln \left( \frac{z+1}{z-1} \right) \]

\[ W = \frac{dF}{dz} = U + \frac{M}{2\pi} \left( \frac{1}{z+1} - \frac{1}{z-1} \right) \]

\[ = U + \frac{M}{2\pi} \cdot \left( \frac{-2}{z^2-1} \right) \]

\[ = U - \frac{2M}{\pi} \cdot \frac{1}{z^2-1} \]
Q.2

Point \( P \) in the \( \Sigma \) plane maps onto \( \alpha \) in the \( \Sigma \) plane. Similarly for \( \mathcal{R} \neq \Sigma \).

The coordinates of \( \alpha \) are given by

\[
x = 2 \Gamma_0 + 2 \frac{c^2}{\Gamma_0}
\]

\[y = 0\]

The coordinates of \( \Sigma \) are given by

\[x = 0\]

\[y = 2 \Gamma_0 - 2 \frac{c^2}{\Gamma_0}\]

Width is 5 times height, so

\[
2 \Gamma_0 + 2 \frac{c^2}{\Gamma_0} = 5 \left(2 \Gamma_0 - 2 \frac{c^2}{\Gamma_0}\right) \implies c = \sqrt{\frac{2}{3}} \Gamma_0
\]
\[ F = U\gamma + \frac{\gamma^2}{\delta} \]

This looks like the function for flow past a cylinder - free stream + doublet. However, there's a small difference: the flow past a cylinder of radius \( r_0 \) is given by:

\[ F = U\gamma + U \frac{r_0^2}{\delta} \]

In our case, \( U \) is missing from the doublet term; we can rewrite as

\[ F = U\gamma + U \frac{r_1^2}{\delta} \]

where \( U r_1^2 = r_0^2 \) - i.e. we have flow past a cylinder with radius

\[ r_1 = \frac{r_0}{\sqrt{U}} \]

Now consider \( F(\gamma(2)) \) - this is also a solution to the Laplace equation - the question is, what does it mean physically?
Streamlines in the $\zeta$ plane will map onto the $\zeta$ plane and they will still be streamlines. We know that one of the streamlines is an ellipse with width equal to 5 times its height. So the flow is some kind of flow past an ellipse. To find out what type of flow this is, we need to see what happens away from the ellipse. As $\zeta \to \infty$, we have:

$$\zeta \approx 2\zeta$$

so

$$F(\zeta(\zeta)) = F(\zeta/2)$$

- this is the flow over an ellipse with free stream velocity $U/2$. 
Q. 3

Let us assume that we have critical flow at the throat. In that case,

\[
\left( \frac{p^*}{p} \right) = \left( \frac{2}{\theta + 1} \right)^{\frac{\gamma}{\gamma-1}} = 0.528
\]

\[p^* = 15.84 \text{ MPa}.\] This is greater than atmospheric pressure, 101.3 kPa, so our assumption was correct.

\[
\frac{T^*}{T_0} = \frac{2}{\theta + 1} \quad \Rightarrow \quad T^* = 244.2^\circ \text{C (pretty cold)}
\]

For a perfect gas, \(c_p = \frac{\gamma}{\gamma - 1} R\)

\[
R = 1.004 \times \frac{0.4}{1.4} = 0.287 \text{ kJ/kg K}
\]

\[
p = \frac{p}{RT} = \frac{15.84 \times 10^6}{287 \times 244.2} = 226 \text{ kg/m}^3
\]

\[T_0 = T_1 + \frac{V_i^2}{2c_p} \quad \Rightarrow \quad V_i = 313 \text{ m/s}
\]

\[
m = \rho VA = 226 \times 313 \times 2 \times 10^{-4} = 14.15 \text{ kg/s.}
\]

Q. 4 Stokes' flow, isn't it? Flow at low \(Re\) is also known as Stokes flow.