Let \( \mathbf{V}_1 \) and \( \mathbf{V}_2 \) be two velocity fields that satisfy \( \nabla \cdot \mathbf{V} = 0 \) and \( \nabla \times \mathbf{V} = 0 \):

\[
\nabla \cdot (\mathbf{V}_1 + \mathbf{V}_2) = \nabla \cdot \mathbf{V}_1 + \nabla \cdot \mathbf{V}_2 = 0
\]

- can do this because dot product is distributive

\[
\nabla \times (\mathbf{V}_1 + \mathbf{V}_2) = \nabla \times \mathbf{V}_1 + \nabla \times \mathbf{V}_2 = 0
\]

- Cross product is distributive too.

Now let \( \phi_1 \) and \( \phi_2 \) be two potential fields that satisfy \( \nabla^2 \phi = 0 \). This implies that \( \nabla \cdot (\nabla \phi) = 0 \) (incompressible)

Also, \( \nabla \times (\nabla \phi) = 0 \) (irrotational)
Now, \[ \nabla \cdot (\nabla (\phi_1 + \phi_2)) \]
\[ = \nabla \cdot (\nabla \phi_1) + \nabla \cdot (\nabla \phi_2) \]
\[ = 0 \quad (\text{gradient is distributive}) \]

\[ \nabla \times (\nabla (\phi_1 + \phi_2)) \]
\[ = \nabla \times \nabla \phi_1 + \nabla \times \nabla \phi_2 \]
\[ = 0 \]

In a 2-D irrotational, incompressible flow,
\[ \nabla^2 \psi = 0 \]

Same arguments apply as for \( \phi \).
\[ F = U z^2 \]
\[ = U (r e^{i\theta})^2 \]
\[ = U r^2 e^{i2\theta} \]
\[ = U r^2 \cos 2\theta + iU r^2 \sin 2\theta \]

\[ \phi = U r^2 \cos 2\theta \]
\[ \psi = U r^2 \sin 2\theta \]

Sketch of streamlines. (Note that \( \psi = 0 \) when \( 2\theta = n\pi \))
\[ W = 2Uz \]
\[ = 2U \cos \theta + i2U \sin \theta = U - iU \]

\[ u = 2U \cos \theta = 2Ux \]
\[ v = -2U \sin \theta = -2Uy \]

Potential lines:

\[ \phi = Ur^2 \cos 2\theta \]

- This is \( \phi \) when \( 2\theta = \pi \phi + \frac{\pi}{2} \)