Report 1 - Tightly Stretched Wire

Develop a one-dimensional finite element program that will read an input file consisting of a) a control section, b) element connectivity, c) nodal coordinates and d) essential boundary conditions and solve the tightly-stretched string problem. Be sure that your code performs a “data echo” of all the input data.

Recall that the governing equation is

\[ T \frac{d^2v}{dx^2} + w(x) = 0, \]  

(1)

where \( v \) is the displacement in the \( y \)-direction, \( T \) is the tension in the string, and \( w(x) \) represents a distributed load on the wire.

Homogeneous essential boundary conditions are prescribed at each end of the domain as shown below \( (v(0) = v(L_1 + L_2) = 0) \).

![Diagram of tightly stretched string problem](image)

Figure 1: Tightly stretched-string problem.

The loading on the wire consists of two parts.

\[ w_1(x) = 10N/m \quad \text{for} \quad 0 \leq x \leq L_1, \]  

(2)

and

\[ w_2(x) = \frac{25(L_1 + L_2 - x)}{L_2} \quad \text{N/m} \quad \text{for} \quad L_1 \leq x \leq L_1 + L_2, \]  

(3)

where \( L_1 = 2m \) and \( L_2 = 4m \). The tension in the wire is \( T = 30kN \).

For this problem, perform two simulations. In the first calculation, use an element length of 0.5m. In the second simulation, use twice the number of elements. Solve the governing equations for the exact solution and present plots of the exact solution and the two finite element solutions on the same axes. (Plots may be machine or hand generated.)
Report Format

This format should be followed for all reports.

1. Title page with your name and date.

2. A statement of the problem that defines the specific problem under consideration. The problem definition includes the governing equations, boundary conditions and variational principle.

3. A description of the finite element program that consists of a brief introduction to the program and a definition of all the input-output variables follows the problem statement. The program description should present the type of element used and highlight any dependencies on linear algebra libraries, etc.

4. An outline of the element layout, boundary conditions, all underlying assumptions associated with the problem.

5. Results and Conclusions. All pertinent results including any graphics/plots should be reported here. Any comparison of results with other solutions should also be presented here. This section should also report the computer time used for each analysis reported. When appropriate, discuss the results.

6. References.

7. Appendix A. All calculations necessary for the analysis should be reported here, e.g., hand calculations of nodal forces, exact solutions, etc.

8. Appendix B:
   - Complete listing of the code.
   - Input (Note all input data must be echoed).
   - Output data