Chapter 6 HW Hints and Answers

Problem 1. We have an armature *voltage-controlled* DC motor driving a load inertia *via* a gear train (note that the Semester Project motor is armature *current* controlled). All necessary numerical parameters are given.

(a) As suggested, let the state vector be $\mathbf{x} \triangleq \begin{bmatrix} i_a & \theta_m & \omega_m \end{bmatrix}^T = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$. The input is applied voltage $e_a \triangleq u$. Note that here we are analyzing the motor alone; the load is not yet connected.

By rearranging the given motor equations, the state equations for the motor are:

$$\frac{di_a}{dt} = -\frac{R_a}{L_a} i_a - \frac{K_b}{L_a}\omega_m + \frac{1}{L_a} e_a \qquad (1)$$

$$\frac{d\theta_m}{dt} = \omega_m \qquad (2)$$

$$\frac{d\omega_m}{dt} = -\frac{B_m}{J_m}\omega_m + \frac{K_t}{J_m} i_a \qquad (3)$$

From equations (1)–(3) you should be able to construct symbolic system and input matrices **A** and **B**. *NOTE:* Everything should be in SI units, that is, mN must be converted to N; also inductance (mH converted to H).

Since we are interested in plotting current i_a (A), motor displacement θ_m (revolutions), and motor speed ω_m (rpm), I suggest constructing an output matrix **C** to produce those outputs (the feedforward matrix **D** is zero). The **C** matrix should produce the three desired outputs (in the desired units), and to do so it will incorporate the conversion factors from rad/s to rpm and rad to revolutions.

Create a MATLAB state-space model using (for example): `>> motor = ss(A,B,C,D);`. I suggest using a time step of 0.001 second, and one way to create a 0.1-second 10V pulse with a total time of 0.2 seconds is:

```
>> dt = 0.001;                     % Time step is 1 msec
>> t = 0:dt:0.2;                   % Create time vector 0.2 seconds in length
>> pulse = ones(101,1)*10;         % Pulse of magnitude 10 of length 0.1
>> input = [pulse;zeros(100,1)];   % Merge pulse and 0.1 second of no input
>> plot(t,input);                  % Plot pulse input just to make sure it's correct!
```

To simulate the system's response to this input, you can use the procedure below:

```
>> motor = ss(A,B,C,D);       % Create SS LTI model of motor
>> y = lsim(motor,input,t);   % Find response to "pulse" input created previously
>> i_a = y(:,1);              % Extract the three columns of the output; i_a (A) is
>> theta_m = y(:,2);          % column 1, theta_m (revolutions) is column 2, and
>> omega_m = y(:,3);          % omega_m (rpm) is column 3.
```

Of course to have the output in the units you want (A, revolutions, rpm) requires the **C** matrix to be constructed properly! The plots should reveal current "spikes" at each end of the pulse, a motor speed which rises to steady-state, then drops to zero when the pulse ends, and a position that increases while the pulse is present.

I found the maximum current to be about 5.4 A.

(b) Now you're adding the load dynamics to the motor. As stated in the problem, when a motor drives a load through a gear train with ratio n ($n > 1$ for speed reduction) the load inertia "referred" to the motor shaft (the usual practice) is the load inertia divided by n^2. I suggest keeping the same state vector as in part (a), but modify the output matrix **C** to produce the desired output (**C** will now contain gear ratio n to convert from motor "space" to load "space" as well as the conversion to degrees).

You will have to compute the mass of the load inertia disk using its dimensions and the mass density of Aluminum ($\rho_{Al} \approx 2.7$ gm/cc).

The motor now has to work a little harder, but the behavior should be similar.

(c) (OPTIONAL) This part does not appear in the notes, but it is instructive. A common assumption is to neglect the armature inductance (let $L_a = 0$); hence the armature voltage equation becomes

$$e_a - i_a R_a - K_b \omega_m = 0.$$

Now there are no "current dynamics" and armature current i_a is no longer a state variable (it may be eliminated using the three motor equations).

Use state vector $\mathbf{x} = \begin{bmatrix} \theta_m & \omega_m \end{bmatrix}^T$ and repeat parts (a) and (b). Are the results much different? Is neglecting the armature inductance a good assumption?

Problem 2. Problem 2 is optional; good luck...