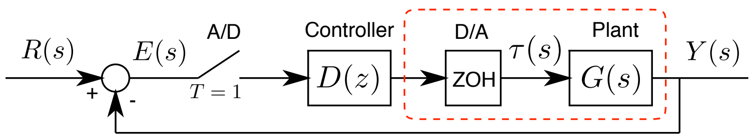
Chapter 5 HW Hints

Problem 1. These should be fairly straightforward using the MATLAB `rlocus` function. All three are already expressed in "root locus" form. Note that you should be able to convert from this form back to the "polynomial" characteristic equation form (and back again).

Problem 2. This is an exercise in controller design in the z-plane. The plant is a simple unit inertia, while the actuator can apply a torque to it. The output is attitude, and you have a measurement of it.

(a) This is simply discretization using the "zero-order hold" method, which we did in Chapter 4. Note that sample period $T = 1$.

(b) The block diagram of the controlled system is shown below.



The $G(z)$ you find in part (a) includes the `ZOH` and the plant $G(s)$. You should find the discrete system transfer function $Y(z)/R(z)$, put its characteristic equation (denominator = 0) into root locus form, and draw the locus. Probably its response will be unsatisfactory (*i.e.* does the locus pass through desirable areas on the z-plane?) Just as for continuous systems, the *type* of a discrete system is the number of "free" integrations around the loop (in the forward path; there are never integrations in the feedback path).

(c) The combination of $\zeta = 0.5$ and $\theta = \pm 45°$ (this is the angle measured from the positive real axis in the standard manner) should allow you to find locations in the z-plane. The lead network needs to draw the root locus to the left such that it passes through those points. The form of the $D(z)$ is

$$D(z) = K\frac{z+b}{z+a},$$

in which the zero should be nearer the +1 point than the pole (this will yield positive phase angle, or phase *lead*). My strategy for a lead network is to place the zero of $D(z)$ at about 1/3 the frequency of the desired closed-loop poles (*NOTE:* consider the "frequency" of a real pole to be its magnitude, *e.g.* a pole at $s = -50$ has a frequency of 50 rad/s), then use the root-locus angle condition to find the pole location of $D(z)$. The zero will be closer to "+1" than the pole. If the pole ends up outside the unit circle (to the left) move the zero to the right and try again. I will be doing this in class with a similar example. If you do this correctly, the modified root locus will pass right through your desired point. You must then use the `rlocfind` function (RL magnitude condition) to obtain the gain K that corresponds to placing the poles at the desired values.

Finally, use the complete compensator $D(z)$ in conjunction with the discretized plant $G(z)$ to find the discrete closed-loop transfer function and plot its step response. You may be surprised at the amount of overshoot, but this is because of the closed-loop zero of $D(z)$, and *is not a bad thing!*. It is much more realistic to examine the damping ratio and natural frequency of the closed-loop transfer function. This can be easily done using the MATLAB `damp(Transfer_Function)` command.

(d) Use the MATLAB `bode()` function to find these frequency responses. Remember that the phase and gain margins are based on the *open-loop* transfer function. Do these look reasonable?

Problem 3. Again design a lead compensator controller for the system of Problem 2, but this time ignore sampling: all design will be in the s-plane. The four steps of Problem 2 will be modified as follows:

(a) Since everything is continuous, this part is unnecessary.

(b) Sketch the root-locus of the uncompensated system in the s-plane. What type is the uncompensated system?

(c) Add a lead network $D(s)$ so the dominant poles are at the s-plane locations **corresponding to** the z-plane locations you used in part 2(c). To have positive phase angle, the zero of $D(s)$ will be closer to the s-plane origin than the pole. You can use the same guidelines in placing the zeros of $D(s)$ as I indicated for Problem 2.

(d) Do the same frequency response plots for the continuous system.

Now, do the following **additional** parts:

(e) Discretize your $D(s)$ using the `trapezoidal (tustin)` method, and use the $G(z)$ from part 2(a) to find the discrete closed-loop system transfer function.

(f) Find the closed-loop poles from (e). Are they where you placed them in Problem 2?

(g) Find the damping ratio and natural frequency (of the dominant closed-loop poles) of both this discrete system and the discrete system from Problem 2.