Chapter 2 HW Hints

Problem 1. Here you're given a difference equation, and asked to do several things.

(a) Express the transfer function $H(z)$ both using "negative" and "positive" powers of z. Apply the $\mathcal{Z}$-transform to each term of the difference equation, where a term that is delayed by one sample (*e.g.* with a "$k-1$" sample index) will have its $\mathcal{Z}$-transformed term multiplied by the unit delay z^{-1}, so $\mathcal{Z}\{y_{k-1}\} \implies z^{-1}Y(z)$. My result for the negative power version is

$$H(z) = \frac{Y(z)}{U(z)} = \frac{0.25z^{-1}}{1 - 0.5z^{-1} - 0.5z^{-2}}$$

(b) Draw a block diagram using only unit delays, gains, and summations. There are many possibilities; I don't care too much which one you find as long as it's correct.

(c) Here is where you check the correctness of your block diagram—it should reduce to yield the transfer function from part (a). Use "standard" block diagram reduction techniques.

Problem 2. (a) Since the "zero point" of the sample index k is arbitrary, I think you are better off using points e_{-1}, e_0, e_1 at times $-T$, 0, and T, respectively. I will try to remember to discuss this problem in class. You should fit the quadratic $\hat{e}(t) = a_0 + a_1 t + a_2 t^2$ through the three points e_{-1}, e_0, e_1 at the corresponding times, solving for the three coefficients of the quadratic. Then integrate this quadratic (don't substitute for the a_i yet—keeps things neater) over the *second half* of its span—from 0 to T. *Then* substitute in for the a_i coefficients you just found and you'll get area A, just as in the trapezoidal integration example. Finally, "generalize" sample indices -1, 0, 1 to $k-2$, $k-1$, k, and you'll have the difference equation for *parabolic integration*, a partial solution is:

$$u_k = u_{k-1} + \frac{T}{12}(5e_k + ...)$$

(b) You should be able to get the transfer function without a problem. Express the transform in positive powers of z with both numerator and denominator polynomials "monic" (coefficient of highest power of z is unity) and a constant term factored out front, like this (*NOTE: this is **not** the answer*):

$$\frac{U(z)}{E(z)} = \frac{T}{4}\left[\frac{z^2 + z + 1}{z(z+3)}\right]$$

In the correct answer, one of the zeros is at $z = 0.1165$.

(c) The exact integral yields a result of $1/\pi = 0.3183$. I assumed initial conditions (samples before $k = 0$) were all zero, and just used a calculator to make a table of all the samples to compare the trapezoidal and parabolic rules. I found both methods to be a little low, but the parabolic was more accurate than trapezoidal, as expected.

Problem 3. Instead of working with the z-plane pole real and imaginary parts a and b, approach this problem by considering the magnitude and angle of the z-plane pole location, that is

$$z = e^{sT} = a \pm jb \implies |z| = \sqrt{a^2 + b^2},\ \angle z = \tan^{-1}\frac{b}{a} = \text{atan2}(b, a)$$

where we use the "atan2()" function to return a "four-quadrant" angle. Expanding the mapping of the s-plane pole to the z-plane using $|z|$ and $\angle z$ should be somewhat simpler. A partial result is:

$$\omega_n = \frac{\sqrt{\ln^2|z| + ...}}{T}$$

Problem 4. Define `LTI` system `G` using the MATLAB `>> G = tf(num,den,T)` function with $T = 0.2$ sec. (a) Use the MATLAB `impulse(G)` function to find (and plot) the impulse response. For part (b) do likewise with the MATLAB `step(G)` function. The response to an arbitrary input (like the sine wave) can be found using `lsim(G,u,t)`, where `u` is the input vector. You should be able to pick off the approximate amplitude ratio and phase shift from the plot—compare these with the analytical calculations. Note that from the sine plot you will get the *time shift*, not the *phase shift*. Time is related to phase by $\phi = \omega t$.