Aliasing and Notch Filter Demonstration

This handout will describe the laboratory setup and material demonstrated we have in MTTC for the demo on Friday, February 22. Please look at this before Friday so you have some idea of what to expect.

1 Aliasing Demo

The purpose of this demonstration is to illustrate that the repeated spectra caused by sampling *really are there.* You will both *see* the aliases and *hear* them. If that isn't enough proof, then I give up.

1.1 Laboratory Setup

The block diagram in Figure 1 below shows a laboratory setup with an A/D converter, computer program which passes the numbers "straight through," and D/A converter (ZOH) at the output. Variables $r(t)$, $r^*(t)$ represent the continuous and sampled inputs, while $r_h(t)$ is the desampled output. The sampling frequency will be 1000 Hz (1 kHz), so sample period $T = 0.001$. The output will be connected to both a spectrum analyzer (1978 edition) and an audio amp (Heathkit tube-type) and a speaker (8Ω impedance).

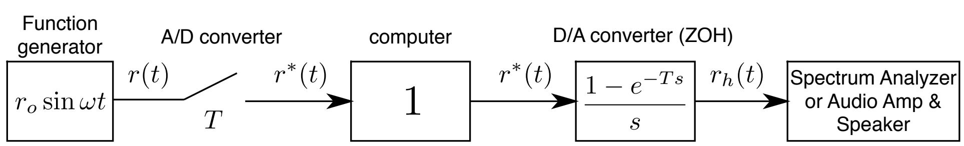

Figure 1: Block diagram of laboratory setup.

1.2 Sinusoidal Input and Frequency Spectrum

A sinusoidal function generator will be used to provide the input, so

$$r(t) = r_o \sin \omega t \tag{1}$$

and the frequency spectrum $R(j\omega)$ will be a single "line" at ω (the spectrum analyzer and function generator actually both have frequency scales in Hz).

1.3 Sampled Output and Frequency Spectra

The frequency spectrum of the D/A output $r_h(t)$ is

$$R_h(j\omega) = R^*(j\omega)H_o(j\omega) \tag{2}$$

with magnitude

$$|R_h(j\omega)| = \frac{1}{T}\sum_{n=-\infty}^{\infty} |R(j\omega - jn\omega_s)|T\left|\text{sinc}\frac{\omega T}{2}\right| \tag{3}$$

The process of sampling will cause the repeated spectra shown as the dotted lines in Figure 2, but their magnitude will be attenuated by the amplitude ratio of the ZOH (red line in Figure 2). The actual spectral lines you will see will those indicated by the solid black lines in Figure 2.

The spectral amplitudes of the continuous input and the sampled and reconstructed output will be displayed on a spectrum analyzer. The expected amplitudes at several frequencies are shown in the accompanying table. Signals at these frequencies will be attenuated accordingly.

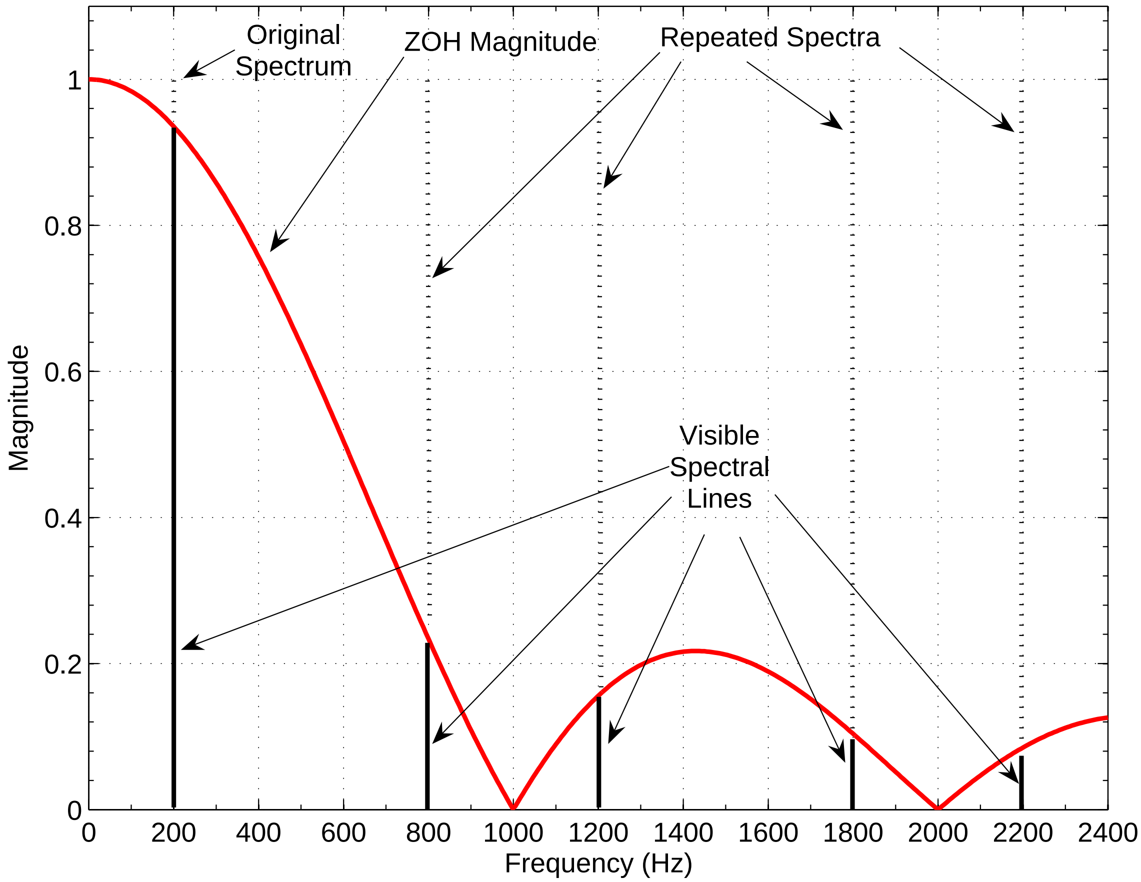

Figure 2: Repeated spectra and attenuation by ZOH.

f (Hz)	ω	$\omega T/2$	$\lvert\text{sinc}(\omega T/2)\rvert$	db
200	1257	0.6283	0.9355	-0.58
800	5027	2.5133	0.2339	-12.62
1200	7540	3.7699	0.1559	-16.14
1800	11310	5.6549	0.1039	-19.66
2200	13823	6.9115	0.0850	-21.41

The magnitude of the ZOH frequency response at a sampling frequency of 1000 Hz is shown in Figure 2. If the input is a sinusoid at 200 Hz, there will repeated spectra at 800, 1200, 1800, 2200…Hz, *etc.* The discrete spectral lines are shown as dashed lines, while their attenuated versions (which are what we actually see in $R_h(j\omega)$ are shown by solid lines.

1.4 When does Aliasing Occur?

This was a question during the demo, and is a very good one. My answer is as follows:

> "Aliasing occurs whenever the signal being sampled has any harmonic content (*i.e.* frequencies) greater than the Nyquist frequency (half the sampling frequency)"

The repeated spectra shown in Figure 2 will **always** occur. The range of frequencies $0 \leq f \leq f_s/2$ (0..500 Hz in Figure 2) is where the "valid" signal is. If the sampled signal has a component Δf above $f_s/2$, that component will *appear* at frequency Δf *below* $f_s/2$. This is aliasing. Most signal processing equipment has analog "anti-aliasing filters" *before* the signal gets sampled. After sampling it's too late.

2 Notch Filter

Recall Chapter 3, HW Problem 3 in which we found the discretization of a notch filter. This lab demo is a good opportunity to see (and hear) an actual notch filter.

2.1 Specific Filter and Prewarped Discretization

Recall that a transfer function for a 2^{nd} order notch filter is

$$G(s) = \frac{s^2 + \omega_o^2}{s^2 + 2\zeta\omega_o s + \omega_o^2} \tag{4}$$

where ω_o is the notch frequency in rad/s.

We will still be sampling at 1000 Hz ($T = 0.001$ sec), and I selected the notch frequency f_o to be 200 Hz (1256.6 rad/s). Hence from (4) the continuous notch filter is

$$G(s) = \frac{s^2 + 1.579e06}{s^2 + 1777s + 1.579e06} \tag{5}$$

The prewarped discretization with f_o as the prewarping frequency is

$$G(z) = \frac{0.5979z^2 - 0.3695z + 0.5979}{z^2 - 0.3695z + 0.1958} = \frac{0.5979(z - 0.3090 \pm j0.9511)}{z - 0.1848 \pm j0.4021} \tag{6}$$

Note the zeros in (6) are right on the unit circle, at angle $\theta = 1.2566$ rad. This angle θ is actually ωT, hence

$$\theta = 1.2566 = \omega T \implies \omega = \frac{1.2566}{T} = 1256.6 \text{ rad/s.} \tag{7}$$

Thus the *transmission zeros* of this filter are at exactly 200 Hz, as desired.

2.2 Frequency Response of Discrete Notch Filter

The MATLAB Bode magnitude plot of this filter is shown in Figure figbodenotch below.

The notch at 200 Hz is apparent, as is the (aliased) notch at 800 Hz. You will hear the notching behavior of this filter (as well as the high whine of the many aliases) on the speaker.

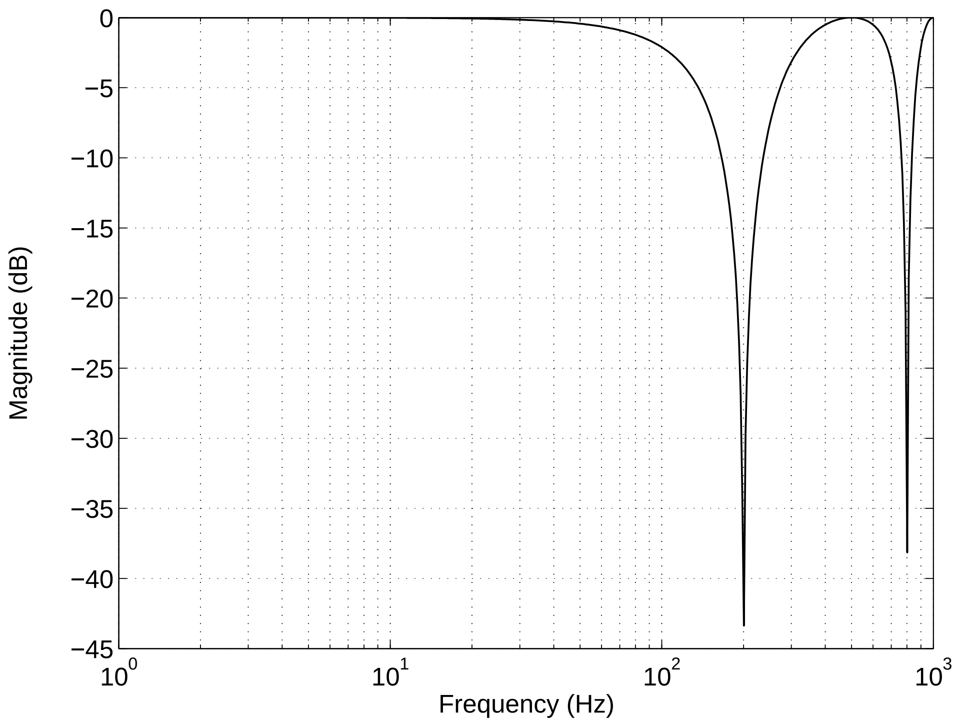

Figure 3: Amplitude Ratio of Notch Filter.