Gear Train Analysis

It is very common to use a *gear train* following a fractional-horsepower motor in servo positioning applications. Small electric motors are usually low-torque and high-speed and a speed-reducing gear train is necessary. A knowledge of the effects of a gear train is important to the control systems designer.

System. Consider a motor driving a rotary load through a gear train, as shown in Figure 1. The angular position of the motor is given by θ_m, while the angular position of the load is given by θ_L. The gear ratio is n, where $n > 1$ for a *speed reducer*. This is the same convention as used in automotive transmissions.

The motor has armature inertia J_m, while the inertia of the load is J_L. There could also be viscous friction acting, but this is not shown for simplicity.

The motor generates torque T_m, which is applied to the load.

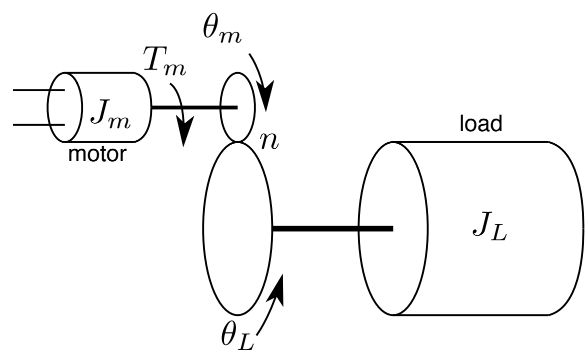

Figure 1: Motor, gear train, and load.

Analysis. The analysis of the gear train is relatively simple, but the result may be unexpected.

Motor angular displacement and load angular displacement (and their derivatives) are related by the gear ratio n:

$$\theta_m = n\theta_L \tag{1}$$

$$\dot{\theta}_m = n\dot{\theta}_L \tag{2}$$

$$\ddot{\theta}_m = n\ddot{\theta}_L \tag{3}$$

Note that for $n > 1$ the gear train is in fact a *speed reducer*.

A speed-reducing gear train is also a *torque multiplier*. The torque delivered by the motor T_m and the torque applied to the load T_L are related by

$$T_L = nT_m \tag{4}$$

Consider the torque balance on the load:

$$T_L = J_L\ddot{\theta}_L \tag{5}$$

Substituting from (3) and (4), we can write (5) as

$$nT_m = \frac{J_L}{n}\ddot{\theta}_m \tag{6}$$

which is rearranged to yield

$$\boxed{T_m = \underbrace{\frac{J_L}{n^2}}_{J_{eff}}\,\ddot{\theta}_m} \tag{7}$$

Equation (7) is important—it is a torque equation in “motor coordinates.” The torque T_m is the motor torque, and the inertia J_{eff} is the effective inertia “felt” by the motor. The load inertia has been reduced by a factor of n^2 due to the gear train.

It is quite common to have gear ratios of 40, 50, or 100. Thus the load inertia may be reduced so much that it is actually less than the motor’s own inertia J_m.

In controls work we usually “refer” a load inertia to the motor shaft, which is what equation (7) shows.

Although not shown here, if the load had a viscous friction coefficient b_L, it would be reduced by a factor of n^2 when “referred” to the motor shaft, so the torque equation would look like

$$\boxed{T_m = \underbrace{\frac{J_L}{n^2}}_{J_{eff}}\,\ddot{\theta}_m + \underbrace{\frac{B_L}{n^2}}_{B_{eff}}\,\dot{\theta}_m} \tag{8}$$