# Chapter 8 HW Assignment & Hints (complete)

**Review Questions.** 1, 4, 5, 7. As usual, I think these are just a matter of text lookup.

**Problems.** 1abegh, 2abc, 8, 30 ~~41~~, 60.

**Problem 1.** Look at RL sketches a, b, e, g, h and, if invalid, list *all* reasons why it is invalid. For example, (a) cannot be invalid because it is not symmetric around the real axis.

**Problem 2.** Do parts a, b, and c. Since you aren't given numbers, all you can do is show the real-axis branches, and approximate asymptote location (intersection and angles). Parts (b) and (c) have departure angles (which you can't compute) so just assume something reasonable.

**Problem 8.** You are given the characteristic equation as a cubic polynomial. Put this in "root locus" form using parameter $K$, then sketch the locus. Find the following "by hand:"

- Real-axis branches
- Asymptote intersection and angle
- Departure angle from complex poles
- Imaginary axis crossing point and corresponding $K$

I found the RL diagram to have 3 poles (2 complex conjugates) and 1 zero. The departure angle from the upper complex pole was between 5° and 15°, and the imaginary axis crossing was between j3 and j4 (corresponding $K$ between 0.2 and 0.3).

**Problem 30.** For this problem, please do the following:

**a.** Do the problem as stated; you may use MATLAB to draw the root locus.

**b.** Find gain $K$ such that the complex closed-loop poles have damping ratio $\zeta \approx 0.5$.

**c.** Using the $K$ from part (**b**), find the **factored form** of the resulting closed-loop transfer function and—using either MATLAB or Simulink —plot the unit step response.

**d.** Is the overshoot what you expect? Is the closed-loop system dominated by a pair of complex poles?

**e.** Is the settling time what the text problem statement requested? Why not?

**f.** Use the root-locus diagram of (**a**) to select a $K$ that should satisfy the settling time requirement.

**g.** Use either MATLAB or Simulink to plot the step response with the $K$ from (**f**). Does this step response look desirable?

**Problem 60.** I had hoped this would be a good problem, but it needs some revision. I modified the block diagram of the system to be more descriptive, as shown below

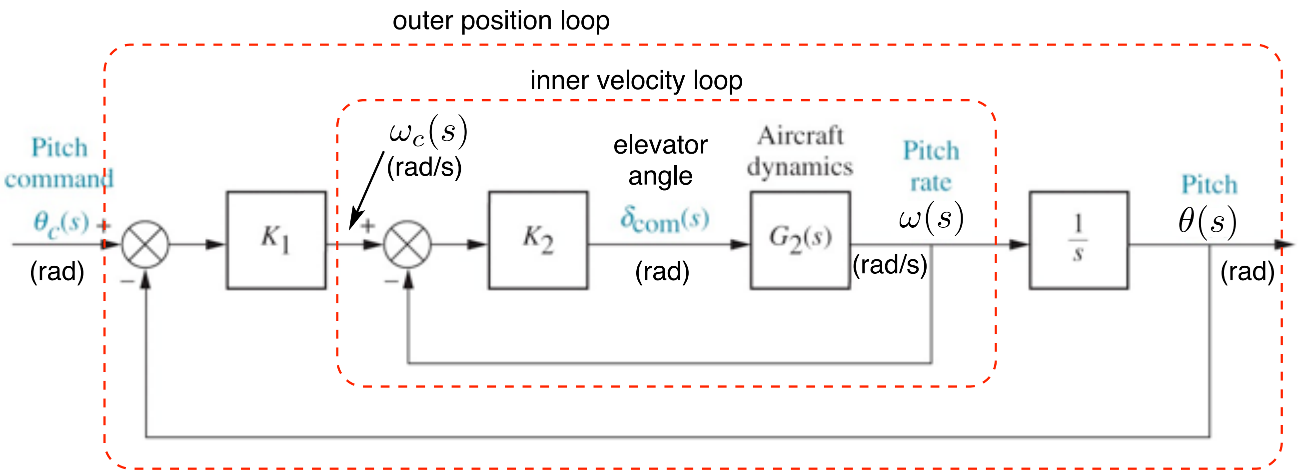
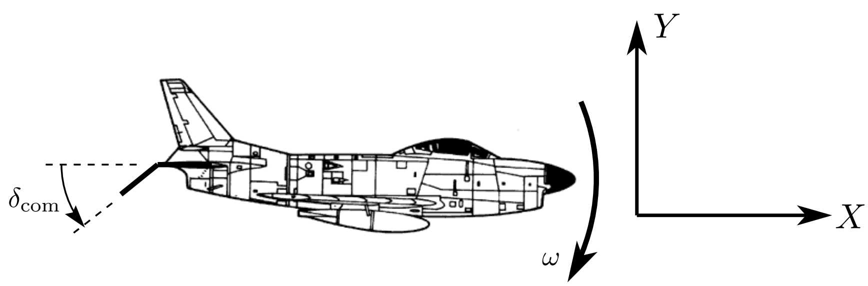



Figure 1: F4E Pitch Stabilization Loop.

Here is how this pitch angle control system is supposed to work:

- Block "Aircraft Dynamics" $G_2(s)$ has input of **elevator angle** $\delta_{com}$, and output **pitch rate** which I have renamed $\omega$. This **pitch rate** is simply the angular velocity of the aircraft as it pitches up/down, and h aas units of rad/s.
- The "inner loop" with gain $K_2$ is intended to control this pitch rate $\omega$. Therefore its input should be commanded pitch rate (rad/s), which I have labeled as $\omega_c$.
- The "outer loop" with gain $K_1$ is intended to control the pitch angle, which I have relabeled as $\theta$.
- This structure of an "inner loop" controlling velocity, and an "outer loop" controlling position is very common in motion control. At least *this* part of the problem was reasonable.

**Behavior of "aircraft dynamics."** Consider the sketch shown below, where the $Z$ axis defines positive angle.




The aircraft elevator is shown with a positive displacement $\delta_{com}$ (positive around $Z$ axis by RH rule). By intuitive reasoning you can see that this elevator displacement will cause the aircraft to pitch **down**, as shown by the angular direction $\omega$.

By the sense of positive angle, this resulting $\omega$ is in the **negative** direction. This is the reason for the "minus" sign in the numerator of "aircraft dynamics" transfer function $G_s(s)$.

**Steps in Problem.** Here is what I want you to do on this problem:

**a. Simulate the behavior of the aircraft dynamics.** Apply a 1° positive elevator angle (*i.e.* downward elevator angular displacement as shown in the sketch). Use MATLAB to plot the pitch rate $\omega$ (rad/s) *vs* time for 1 second. What is the pitch rate (rad/s) after 1 second? Does this seem reasonable?

**b. Inner loop CL TF.** Show that the closed-loop transfer function of the inner loop is

$$\frac{\omega(s)}{\omega_c(s)} = \frac{-508K_2(s+1.6)}{(s+4.9)(s+14)(s-1.8) - 508K_2(s+1.6)} \; \frac{\text{rad/s}}{\text{rad/s}} \tag{1}$$

**c. Inner loop RL form.** Show that a **root-locus** form of the inner-loop characteristic equation of (1) is

$$1 - 508K_2 \frac{s+1.6}{(s+4.9)(s+14)(s-1.8)} \tag{2}$$

**d. Selection of inner loop gain.** Define $K \triangleq -508K_2$, thus you will now have an inner loop RL form of

$$1 + K\frac{s+1.6}{(s+4.9)(s+14)(s-1.8)} \tag{3}$$

Use MATLAB to draw the root locus diagram of (3), and select $K_2$ (actually $K$) to place the complex poles to yield a damping ratio of $\zeta = 0.5$. I found that $-0.4 > K_2 > -0.5$.

**e. Factored form of inner loop TF.** With the value of $K_2$ from part (**d**), find the **factored form** of inner loop transfer function $\dfrac{\omega(s)}{\omega_c(s)}$. This will be the "middle" block in the forward path of the outer loop.

**f. Response of controlled inner loop.** Use either MATLAB or Simulink to find the step response of the controlled inner loop to a commanded pitch rate of $\omega_c = 1°/\text{sec}$. Plot the resulting pitch rate $\omega$ (degrees/sec) versus time. Does it look reasonable? What is the final value of $\omega$ in degrees/sec? Does this agree with the DC gain of the inner loop?

**g. Block diagram of outer loop.** Using the factored form of the inner loop from part (**e**), draw a **block diagram** of the outer loop (similar to Figure 1 on the previous page). The outer loop forward path will have three blocks in series: (1) gain $K_1$, (2) factored form of inner loop TF from part (**e**), and (3) the integrator $1/s$. What is the **TYPE** of the outer loop system?

**h. Closed-loop TF of outer loop.** From the block diagram of part (**g**), find the closed-loop transfer function $\dfrac{\theta(s)}{\theta_c(s)}$.

**i. Outer loop CE and RL form.** Find the characteristic equation of the outer loop, and put it in root-locus form using parameter $K_1$.

**j. Selection of outer loop gain.** Use MATLAB to draw the root-locus diagram of part (**i**) versus gain $K_1$. Select $K_1$ so the complex poles have a damping ratio of $\zeta = 0.45$.

**k. Factored form of outer loop.** With the value of $K_1$ you selected in part (**j**), what is the **factored form** of the outer-loop transfer function (the entire system)? This will be $\dfrac{\theta(s)}{\theta_c(s)}$.

**l. Response of controlled outer loop.** Using Simulink , construct a model of the entire system. You will have the two nested loops. Use the two numerical values you've found for $K_2$ and $K_1$, plus all other numerical values, of course.

Let the commanded pitch angle input be a step of 1°, and produce the following two plots:

- Plot the actual pitch angle $\theta$ (degrees) of the aircraft versus time
- Plot the elevator angle $\delta_{\text{com}}$ (degrees) versus time

Does the response of the aircraft look reasonable? Does the response of the elevator angle look reasonable (you may have to think about that a little)?

This problem is a lot of work, but I've tried to lead you through all the steps methodically...