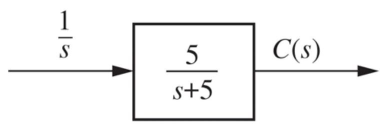
Chapter 4 HW Assignment & Hints

Review Questions. 1, 2, 5, 6, 8, 9, 10, 13, 14. As usual, I think these are just a matter of text lookup.

Problem 2(a). Just multiply the transfer function by the input to get output $C(s)$, then inverse transform $C(s)$ (use table) to get $c(t)$. Time constant is easy to see; use corresponding equations for rise time and 2% settling time.



Problem 3(a). Same system as above, but use MATLAB `step` function to find step response. I'd suggest doing something like:

```
>> numG = [xx xx]; % Define TF numerator
>> denG = [xx xx]; % Define TF denominator
>> G = tf(num,den); % Define transfer function
>> [y,t] = step(G); % Find step response
>> plot(t,y); % Plot step response
```

Make sure your plot has labeled axes (we are not given the units of the output).

Problem 8. You are just to do items (b) and (d). The poles and zeros are easy to find. The "general form of the step response" should just have the "form" of the response modes, but not the weighting.

For example, for part (c) the form of the step response would be

$$y(t) = A + Be^{-10t} + Ce^{-20t}$$

The step response will always include a constant term (A in the above example) plus the transient terms. You can use the Laplace Transform table I have on the class website to do these if you wish.

Problem 9. Since the poles are just roots of the denominator polynomial, you can use the MATLAB `roots` function to find these poles. For example, to find the roots of polynomial $s^3 + 12s^2 + 47s + 60$ you can just enter the coefficients, then use the `roots` function:

```
>> c = [1 12 47 60];
>> roots(c)

ans = -5.0000
      -4.0000
      -3.0000
```

So the roots of the above example polynomial are $s = -3, -4, -5$. I believe problem 9 has some complex conjugate roots.

Problem 18. The standard form of a second-order transfer function denominator is

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

By equating coefficients and solving for damping ratio ζ and (undamped) natural frequency ω_n you should be able to answer this problem.

Problem 20(c). Similar approach to the previous problem. I obtained a natural frequency a little over 500 Hz (remember ω_n is in rad/s).

Problem 21(c). Use the MATLAB `step` function and plot the response as before. From the problem statement, your MATLAB script is supposed to compute everything except rise time t_r, which you should get from the plot. That makes sense, since all the other performance specifications have simple expressions, but rise time t_r really does not (actually there *ARE* approximations, but this text doesn't show them).

Problem 23. This is the reverse problem to that of Problem 20; it is more of a design problem. Given some time-domain response specs, find the corresponding pole locations.

I suggest finding ζ and ω_n from the given time-response specs, then you know the location of the (underdamped) poles are just

$$s = -\zeta\omega_n \pm j\omega_d = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

I'm assuming that none of the given specs yield overdamped behavior (unlikely).

Problem 29(c). From the height of the response peak you can estimate damping ratio ζ, and from the peak time you can estimate damped frequency ω_d. From this you should be able to estimate the transfer function.

Problem 30. This problem is **poorly posed**!! In the first line, change the word "response" to the word "transfer." Then remove the "s" from the denominator of each function. Finally, rename each $C(s)$ to $G(s)$ (a better letter to use for a transfer function). **Then** you can compare the responses.

NOTE: *DO NOT* bother to compute the %OS, settling time, *etc.* You've done that enough.

Just use MATLAB to find and plot the unit step response of each original system plus that of the "cancelled" system on the same plot (four plots: two responses on each plot). Adjust the numerator coefficient of the "cancelled" system so the "DC Gain" of the "cancelled" system is the same as the original.

For example, for part (a), the "original" and "cancelled" transfer functions are:

$$\text{original } G(s) = \frac{(s+3)}{(s+2)(s^2+3s+10)} \quad \text{(dc gain 3/20)}$$

$$\text{cancelled } G(s) = \frac{1.5}{s^2+3s+10} \quad \text{(dc gain 1.5/10 = 3/20)}$$

The numerator value of 1.5 was selected so the DC gains of both transfer functions are the same (recall to find DC gain just let $s = 0$ in the transfer function).

DC Motor Problem. Use the system of part (c) in the Chapter 3 HW assignment, and find transfer function $G(s)$, where

$$G(s) = \frac{\theta_L(s)}{E_a(s)} \quad \frac{\text{rad}}{\text{V}}$$

Plot the response of θ_L (rad) to a 10V step input in motor voltage e_a. Use MATLAB, and plot for 0.1 second.