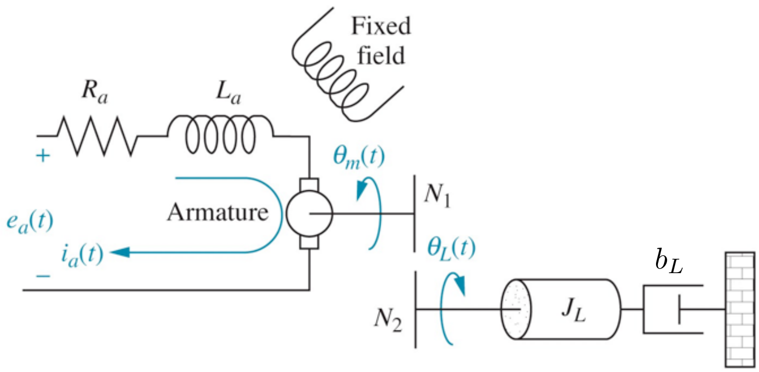
Chapter 3 HW Assignment & Hints

Problem 19. Consider a DC motor driving a load through a gear train:



The specifications for the Pittman 7214 DC motor (taken from the manufacturer's data sheet) are:

$$J_m = 1.54 * 10^{-6} \text{ kg-m}^2 \quad \text{(not shown in figure)}$$
$$b_m = 2.7e-3 \ \frac{\text{mN-m}}{\text{rad/s}} \quad \text{(not shown in figure)}$$
$$L_a = 0.69 \text{ mH}$$
$$R_a = 1.53 \ \Omega$$
$$K_t = 22.3 \ \frac{\text{mN-m}}{\text{A}}$$
$$K_b = 0.0223 \ \frac{\text{V}}{\text{rad/s}}$$

The load inertia is a disk made of aluminum with the following dimensions:

$$\text{radius } r = 100 \text{ mm}$$
$$\text{thickness } t = 10 \text{ mm}$$
$$\text{viscous damping } b_L = 4 \ \frac{\text{mN-m}}{\text{rad/s}}$$
$$\text{gear ratio } n = \frac{N_2}{N_1} = 50$$

(a) Choosing the state variables to be $x_1 = i_a$, $x_2 = \theta_m$, and $x_3 = \dot{\theta}_m = \omega_m$, and input $u = e_a$, write the symbolic (letters, not numbers) state equations and find the symbolic **A B C D** matrices for the motor alone (do **not** neglect armature inductance L_a). The output matrix **C** should produce an output vector (three different outputs) of armature current (A), motor displacement (revolutions), and motor velocity (rpm).

Simulate the motor response to a voltage pulse of magnitude 10V and duration 0.1 second. Simulate for a duration of 0.2 seconds. I suggest using a simulation time step of 1 msec (0.001 sec). Plot motor armature current (A), motor displacement (revolutions), and motor velocity (rpm) *vs* time. What is the maximum armature current? How many revolutions did the motor rotate? (*Answer:* nearly 6A and a little over 7 revolutions)

(b) Repeat part (a) but now **neglect** the armature inductance. You will now have only two state variables: $x_1 = \theta_m$ and $x_2 = \omega_m$. You will have the same three outputs. Is there much difference in the response of the motor compared to (a)? (*Hint:* in your modeling, eliminate i_a from the motor equations to get the two state equations)

NOTE: Since armature current i_a is no longer a state variable (but is output y_1), you can compute it in the output equation $\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}u$ (use the armature voltage equation).

(c) Now add the dynamics of the load (load inertia and damping) to the model of part (b), but ignore the inertia of the gears (it was not given anyway!). Perform the same simulation as in (a). Plot motor current (A), load displacement (degrees), and load velocity (degrees/sec) *vs* time.