Chapter 2 HW Hints

Problem 8a. For the following transfer function, write the corresponding differential equation.

$$\frac{X(s)}{F(s)} = \frac{7}{s^2 + 5s + 10} \tag{1}$$

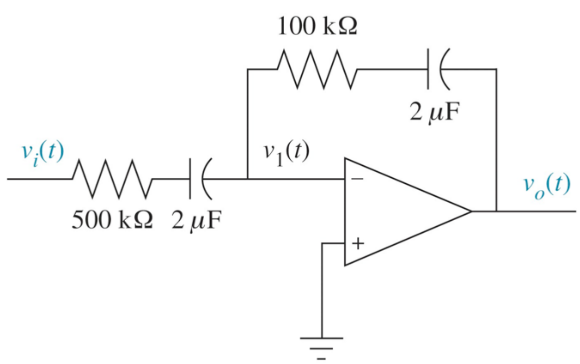
You can simply "cross-multiply" then use the Laplace transform "differentiation theorem," which essentially equates multiplication by s with time differentiation.

The differential equation should come right out.

Problem 21a. This op-amp circuit is *structurally* exactly the same as the op-amp "inverting amplifier" I analyzed in class, which has transfer function

$$\frac{V_o(s)}{V_1(s)} = -\frac{R_2}{R_1} \tag{2}$$

The difference is that Problem 21a has input impedance $Z_1(s)$ (composed of input resistor and capacitor) and feedback impedance $Z_2(s)$ (composed of feedback resistor and capacitor) instead of simply input and feedback resistors R_1 and R_2.



So the transfer function of this problem will be of the form

$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_f(s)}{Z_i(s)}, \tag{3}$$

where

$$Z_i(s) = \text{combined impedance of input components } R_i \text{ and } C_i$$
$$Z_f(s) = \text{combined impedance of feedback components } R_f \text{ and } C_f$$

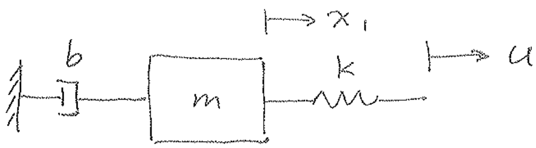
Electrical impedances in series behave just like resistors in series (they add), so the result shouldn't be too hard to find. For example, input impedance $Z_i(s)$ is

$$Z_i(s) = R_i + \frac{1}{C_i s} = \frac{s+1}{(2e-6)s} \tag{4}$$

where the number "2e -6" is just "computer shorthand" for $2 * 10^{-6}$.

Problem 23. I changed the input to this problem: instead of force $f(t)$ applied to the end of the spring, the input is **displacement** $u(t)$ of the free end of the spring (directed positive to the right like displacement x_1). This is much more realistic and easier to analyze.

Thus the problem sketch is like this, where I've define parameters m, b, and k so you can first analyze the problem symbolically (always a good idea):

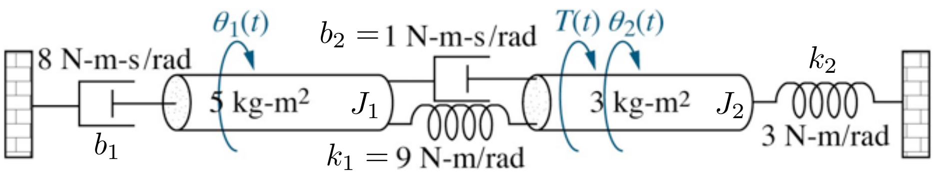


You should get the following equation of motion:

$$m\ddot{x}_1 + b\dot{x}_1 + kx_1 = ku \tag{5}$$

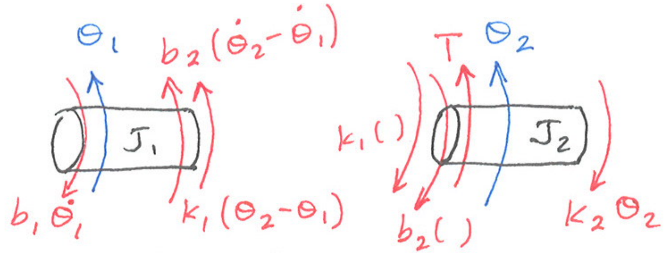
From (5) the transfer function $X_1(s)/U(s)$ should be easy to find. Be sure to put units on the resulting transfer function (again, always a good idea).

Problem 30a. My main complaint with this problem is that the author didn't label any of the components with "symbolic" names (which you will then use in your analysis). So I modified the problem sketch:



Please use inertia, damping, and stiffness parameters $J_1, J_2, b_1, b_2, k_1, k_2$ in your analysis.

For purposes of visualizing the torque directions for the free-body diagram, I assumed that $\theta_2 > \theta_1$, and $\dot{\theta}_2 > \dot{\theta}_1$ (you could assume the opposite). My free-body diagram is shown below:

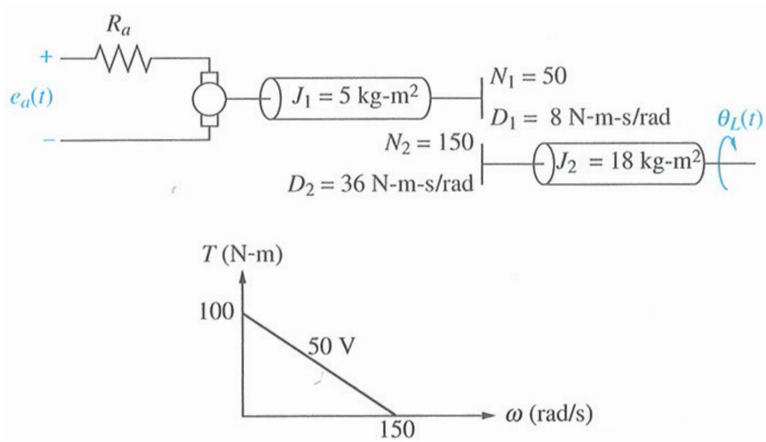


The two moments on J_2 labeled $k_1()$ and $b_2()$ are just $k_1(\theta_2 - \theta_1)$ and $b_2(\dot{\theta}_2 - \dot{\theta}_1)$...I didn't feel like writing them twice.

The equation of motion I obtained for the second inertia was:

$$J_2\ddot{\theta}_2 + b_2\dot{\theta}_2 + (k_1 + k_2)\,\theta_2 - b_2\dot{\theta}_1 - k_1\theta_1 = T \tag{6}$$

Problem 42. This is a DC motor driving a load inertia *via* a gear train. In keeping with my classroom analysis, I used the following terminology:



In keeping with my classroom analysis, I used the following terminology:

$$\begin{aligned} J_m &= J_1 = \text{motor inertia} \\ b_m &= D_1 = \text{motor viscous friction (damping)} \\ J_L &= J_2 = \text{load inertia} \\ b_L &= D_2 = \text{load viscous friction} \\ b_{eq} &= \text{combined motor inertia and back EMF} \\ n &= \frac{N_2}{N_1} = \text{gear ratio} \end{aligned}$$

The total inertia the motor "feels" is J_t, and is given by

$$J_t = J_m + \frac{J_L}{n^2} \tag{7}$$

Likewise the total viscous friction the motor feels is

$$b_t = b_{eq} + \frac{b_L}{n^2} \tag{8}$$

where as you recall the "equivalent" damping coefficient is given by

$$b_{eq} = b_m + \frac{K_t K_b}{R_a} \tag{9}$$

Now, the transfer function from applied motor armature voltage $E_a(s)$ to *motor* position $\theta_m(s)$ is given by

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_a J_t)}{s\left(s + \frac{b_t}{J_t}\right)} \tag{10}$$

where we have used the total inertia and damping (motor + load).

Speed-Torque Equation. Motor parameters K_t, K_b, R_a must be obtained from the given "speed-torque" curve. Remember that $K_t = K_b$ in equivalent units.

Also remember that you have to use the gear ratio n to get the final result in terms of load rotation angle θ_L.

A partial final result is

$$\boxed{\frac{\theta_L(s)}{E_a(s)} = \frac{0.0952}{s(s+???)} \ \frac{\text{rad}}{\text{V}}} \tag{11}$$