

Chapter 7 & 9 HW Solution

Problem 7.1: The center-to-center distance is the sum of the two pitch circle radii. To mesh, the gears must have the same diametral pitch. These two facts are enough to solve for the diametral pitch P . Like this:

$$R_2 + R_3 = 3.625 \text{ in} \quad (1)$$

Since diametral pitch $P = \frac{N}{2R}$ must be the *same* for the gears to mesh, then

$$\frac{N_2}{R_2} = \frac{N_3}{R_3} \quad (2)$$

Solving equations (1) and (2) simultaneously yields

$$R_2 = 1.000 \text{ in} \quad (3)$$

$$R_3 = 2.625 \text{ in} \quad (4)$$

and the quantity that the problem requested, diametral pitch, is

$$\boxed{P = 16} \quad (5)$$

Problem 7.2: Here the pitch diameter is 6 in, so pitch radius $R = 3$ in. Since diametral pitch $P = 9$ teeth/in, then the number of teeth is

$$\boxed{N = 2RP = 2(3)(9) = 54 \text{ teeth}} \quad (6)$$

The **circular pitch** p is the distance from one tooth to the next, and is

$$\boxed{p = \frac{2\pi R}{N} = \frac{2\pi(3)}{54} = \frac{\pi}{9} = 0.3491 \text{ in (per tooth)}} \quad (7)$$

So measured along the pitch circumference, there are approximately 3 teeth/in; quite different from the **diametral pitch** $P = 9$ teeth/in. The difference is a factor of π .

Problem 7.6: The pitch diameter D is simply twice the pitch radius R . Since we know that $P = \frac{N}{2R}$, then

$$N = 2RP = DP, \quad (8)$$

where the number of teeth N must be an integer.

Since $D_2 = 3.50$ in, and $D_3 = 8.25$ in, and the pitch $P = 16$ teeth/in, we have

$$\boxed{N_2 = 3.5 \times 16 = 56 \text{ teeth}} \quad (9)$$

$$\boxed{N_3 = 8.25 \times 16 = 132 \text{ teeth}} \quad (10)$$

Problem 7.13: A $P = 4$ -tooth/in, $N_2 = 24$ -tooth pinion is to drive a $N_3 = 36$ -tooth gear. The gears are formed using the 20° full-depth involute system. Find the following:

- Addendum
- Dedendum
- Clearance

- Circular pitch
- Base pitch
- Tooth thickness
- Base circle radii
- Lengths of paths of approach & recess
- Contact ratio

You have to search through Chapter 7 to find what all these are and how to calculate them. Here's what you get...

- (a) **Addendum:** The addendum a is the radial distance between the pitch circle and top land (face) of each tooth. From Table 7.2—for the 20° full-depth involute system—the addendum a is

$$a = \frac{1}{P} = \frac{1}{4} = 0.250 \text{ in} \quad (11)$$

- (b) **Dedendum:** The dedendum d is the radial distance between the pitch circle and bottom land of each tooth. As before, from Table 7.2 the dedendum d is

$$d = \frac{1.25}{P} = \frac{1.25}{4} = \frac{5}{16} = 0.3125 \text{ in} \quad (12)$$

- (c) **Clearance:** The clearance c is the amount by which the dedendum of a gear exceeds the addendum of the mating gear (text p. 333), so

$$c = d - a = \frac{5}{16} - \frac{1}{4} = \frac{1}{16} = 0.0625 \text{ in} \quad (13)$$

- (d) **Circular pitch:** The circular pitch is the distance between teeth along the pitch circle, and is

$$p = \frac{2\pi R}{N} = \frac{2\pi(3)}{24} = \frac{\pi}{4} = 0.7854 \text{ in} \quad (14)$$

- (e) **Base pitch:** This is the distance between teeth along the *line of action*, and with pressure angle ϕ it's given by

$$p_b = p \cos \phi = \frac{\pi}{4} \cos 20^\circ = 0.7380 \text{ in} \quad (15)$$

Note that base pitch $p_b < p$.

- (f) **Tooth thickness:** The tooth thickness t is simply half of the tooth spacing; where the tooth spacing is the circular pitch p , so

$$t = \frac{p}{2} = \frac{\pi}{8} = 0.3927 \text{ in} \quad (16)$$

- (g) **Base circle radii:** The base circle is the circle from which the involute tooth profile is generated, and can be larger or smaller than the dedendum circle.

From Example 7.2 and Figure 7.23, plus text equation (7.15),

$$r_b = r_P \cos \phi = R \cos \phi \quad (17)$$

Since diametral pitch $P = \frac{N}{2R}$, the radius of the pitch circle is just $R = \frac{N}{2P}$. Therefore the two pitch circle radii are

$$R_2 = \frac{N_2}{2P} = \frac{24}{2(4)} = 3 \text{ in} \quad (18)$$

$$R_3 = \frac{N_3}{2P} = \frac{36}{2(4)} = 4.5 \text{ in} \quad (19)$$

and the corresponding base circle radii are

$$(r_b)_2 = R_2 \cos 20^\circ = 2.8191 \text{ in} \quad (20)$$

$$(r_b)_3 = R_3 \cos 20^\circ = 4.2286 \text{ in} \quad (21)$$

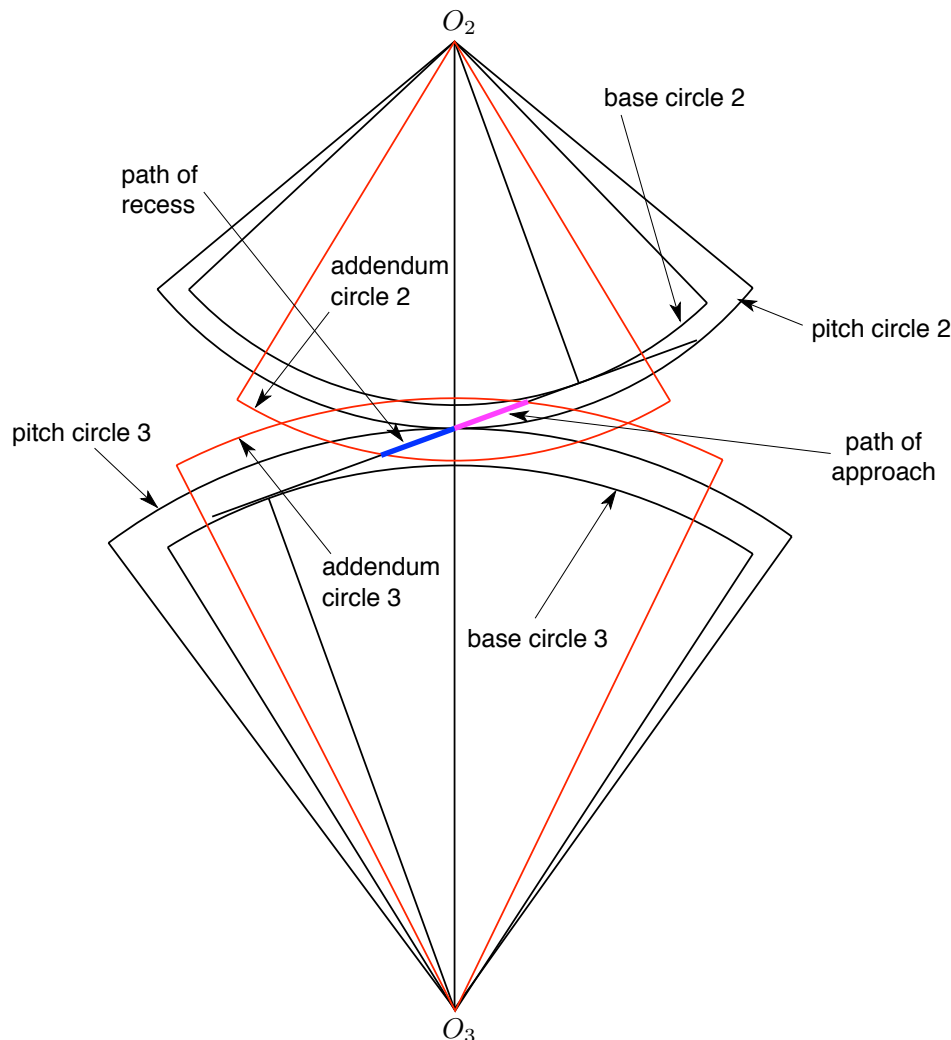
- (h) **Lengths of paths of approach & recess:** The *approach* is the distance (measured along the line of action) from the *beginning* of tooth contact (intersection of addendum circle of gear 3 and line of action) to the pitch point. This is line segment CP in Figure 7.9. In Section 7.8 there is a geometrical equation for CP based on the addendum a , pressure angle ϕ , and pitch radius R , which is

$$CP = \sqrt{(R_3 + a)^2 - (R_3 \cos \phi)^2} - R_3 \sin \phi = 0.6425 \text{ in (length of approach)} \quad (22)$$

Likewise, the *recess* is the distance (measured along the line of action) from the *end* of tooth contact (intersection of addendum circle of pinion 2 and line of action) to the pitch point. This is line segment PD in Figure 7.9. There is a similar equation,

$$PD = \sqrt{(R_2 + a)^2 - (R_2 \cos \phi)^2} - R_2 \sin \phi = 0.5911 \text{ in (length of recess)} \quad (23)$$

As stated in the **HW HINTS**, I wanted you to make a drawing illustrating the **approach** and **recess**; such a drawing is shown below (scaled down to fit the page). The blue and magenta lines are the paths of the contact point from the beginning of contact to the end of contact (approach and recess). You may recall that I showed an animation like this...

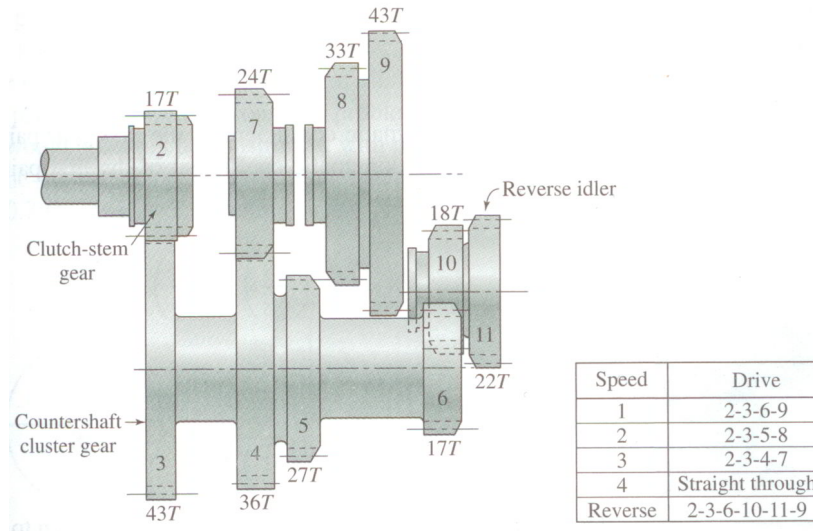


- (i) **Contact ratio:** The contact ratio m_c represents the ratio of the distance over which teeth are in contact (lengths $CP + PD = CD$ in Figure 7.9) to the spacing between teeth along the line of action (base pitch p_b). It represents the “average” number of teeth in contact, and must be greater than one.

In this case

$$m_c = \frac{CD}{p_b} = \frac{0.6245 + 0.5911}{0.7380} = 1.6472 \text{ (avg. teeth in contact)} \quad (24)$$

Problem 9.4: The truck transmission is shown below.



The input is the “clutch-stem gear” and the output is the shaft with gears 7, 8, 9. All gears on the countershaft are connected together.

- (a) **Speed 1:** The drive is through gears 2-3-6-9 (gears 3 and 6 spin at the same rate), so the gear ratio is

$$n_1 = \frac{43}{17} \frac{43}{17} = 6.40 \quad (25)$$

which is a *very* low (*i.e.* a “granny”) first gear.

- (b) **Speed 2:** The drive is through gears 2-3-5-8 (gears 3 and 5 spin at the same rate), so the gear ratio is

$$n_2 = \frac{43}{17} \frac{33}{27} = 3.09 \quad (26)$$

This is more like a “normal” first gear (still pretty low)

- (c) **Speed 3:** The drive is through gears 2-3-4-7 (gears 3 and 4 spin at the same rate), so the gear ratio is

$$n_3 = \frac{43}{17} \frac{24}{36} = 1.69 \quad (27)$$

- (d) **Speed 4:** By “straight-through” they mean the input and output are directly connected, so

$$n_4 = 1.00 \quad (28)$$

- (e) **Reverse:** This is a mistake in the figure! The idea is that when in “reverse” the assembly of gears 10 and 11 slides to the left, where gear 11 engages gear 6, and gear 10 engages gear 9. Hence the “drive” should be: 2-3-6-11-10-9, which will be

$$n_R = \frac{43}{17} \frac{22}{17} \frac{43}{18} = -7.82 \quad (29)$$

This transmission is like the one in my old 1950 Chevrolet pickup, a three-speed with a “granny” first gear (and reverse).

Starr Problem 1: Perform the exercise “suggested” in the last sentence on text page 391, *i.e.* “As an exercise, it is suggested that you seek out a suitable set of (U.S. Customary) diametral pitches for each pair of gears shown in Fig. 9.4 so that the first and last gears will have the same axis of rotation with all gears properly engaged.”

NOTE: Gear 3 should have 42 teeth, not 22 teeth...

There are two conditions which must be met in this situation:

- For two gears to mesh, they must have the same pitch, so $P_2 = P_3 \triangleq P_a$, and $P_4 = P_5 \triangleq P_b$ (the pitch of gears 2 and 3 do NOT need to equal that of gears 4 and 5).
- The distance between the axes of gears 2 and 3 must equal that between gears 4 and 5 (so the axes of input and output shafts are collinear)

The equation for the diametral pitch P can be solved for the pitch radius R :

$$P = \frac{N}{2R} \implies R = \frac{N}{2P} \quad (30)$$

where N = number of teeth, and R = radius of pitch circle (equivalent rolling circle).

We are given the following numbers of teeth for each gear as:

$$N_2 = 18 \quad N_3 = 42 \quad N_4 = 16 \quad N_5 = 24$$

For input/output shaft alignment we want

$$R_2 + R_3 = R_4 + R_5, \quad (31)$$

and substituting (30) into (31) yields

$$\frac{N_2}{2P_a} + \frac{N_3}{2P_a} = \frac{N_4}{2P_b} + \frac{N_5}{2P_b} \implies \frac{N_2 + N_3}{P_a} = \frac{N_4 + N_5}{P_b} \quad (32)$$

where again P_a is the pitch of gears 2 & 3 and P_b is the pitch of gears 4 & 5.

Substituting for the numbers of gear teeth, equation (31) results in

$$\frac{P_b}{P_a} = \frac{N_4 + N_5}{N_2 + N_3} = \frac{40}{60} = \frac{2}{3} \quad (33)$$

Thus the defining relationship between the two pitches is

$$\boxed{3P_b = 2P_a} \quad (34)$$

From Table 7.1 there are **ELEVEN** possible pitch combinations which satisfy (33) are (I think we found them all):

P_a	1 $\frac{1}{2}$	2 $\frac{1}{4}$	3	6	12	18	24	48	72	96	120
P_b	1	1 $\frac{1}{2}$	2	4	8	12	16	32	48	64	80

Starr Problem 2: Design an overdrive unit similar to Example 9.9 so that the percentage reduction in engine speed is 25% and the ring gear has a pitch diameter of approximately 6 inches.

The general expression for the angular velocities in a basic planetary gear train are:

$$\omega_S = -\alpha \omega_R + (1 + \alpha) \omega_C \quad (35)$$

where $\alpha = \frac{R}{S}$ (R and S are numbers of ring and sun gear teeth). The operation of the overdrive is like Example 9.9, with the sun gear fixed, so for a 25% overdrive we need

$$\frac{\omega_C}{\omega_R} = \frac{\alpha}{1 + \alpha} = 0.75 \implies \alpha = 3 = \frac{R}{S} = \frac{N_R}{N_S} \quad (36)$$

- (a) Since we're looking at a ring gear pitch diameter of around 6 inches, I let the diametral pitch $P = 10$, then the ring gear will have

$$N_R = 2R_R P = 2(3)(10) = 60 \text{ teeth} \quad (37)$$

This means the sun gear must have $N_S = 20$ teeth, so its pitch radius is

$$R_S = \frac{N_S}{2P} = \frac{20}{20} = 1 \text{ in} \quad (38)$$

For geometric compatibility the planet gears also have a pitch radius of 1 in, and 20 teeth.

- (b) A full-size drawing is shown on the next page (wouldn't fit below)
- (c) Assuming the 20° full-depth AGMA tooth standard, I'll find the contact ratio between the equal-sized sun and planet gears. We need the "approach" and "recess" distances as in Problem 7.13. So we also need the addendum distance a (we already have the pitch radii) and the base pitch p_b .

From Table 7.1, the addendum is $a = \frac{1}{P} = \frac{1}{10} = 0.10$ in.

Since the gears are the same size, the approach and recess will be equal, and from text equation (7.10) I found

$$CP = 0.2298 \text{ in} \quad (39)$$

From text equations (7.4) and (7.6) the base pitch p_b is

$$p_b = p \cos \phi = \frac{\pi}{P} \cos \phi = 0.2952 \quad (40)$$

So the contact ratio m_c is

$$m_c = \frac{CP + CP}{p_b} = \frac{0.4596}{0.2952} = 1.5569, \quad (41)$$

so a little over 1.5 teeth are in contact all the time.

I didn't look at the planet/ring mesh.

