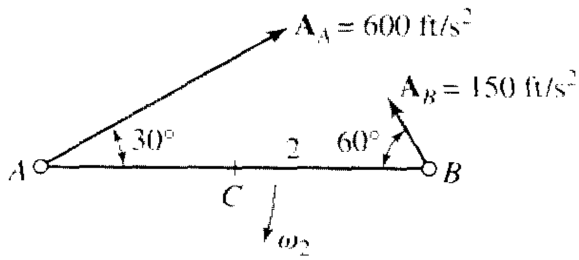
Chapter 4 HW Solution

Problem 4.6: (Analytical Solution Only) The problem is shown below, where we wish to find angular velocity ω_2 and acceleration α_2 and the acceleration of midpoint C (length $r_{BA} = 20$ in):



Since points A, B, and C are all on body 2, we can solve this problem exclusively using the "2 Pts. on a Body" equation. Since the acceleration of A and B is given, use

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{BA} + \boldsymbol{\omega}_2 \times (\boldsymbol{\omega}_2 \times \mathbf{r}_{BA}) \tag{1}$$

where

$$\mathbf{a}_B = -75\mathbf{i} + 129.9\mathbf{j} \text{ ft/s}^2 \tag{2}$$
$$\mathbf{a}_A = 519.6\mathbf{i} + 300\mathbf{j} \text{ ft/s}^2 \tag{3}$$
$$\boldsymbol{\omega}_2 = \omega_2\mathbf{k} \text{ rad/s} \tag{4}$$
$$\boldsymbol{\alpha}_2 = \alpha 2\mathbf{k} \text{ rad/s}^2 \tag{5}$$
$$\mathbf{r}_{BA} = 1.667\mathbf{i} \text{ ft} \tag{6}$$

Substituting (2)–(6) into (1), performing the cross products, and separating **i** and **j**, we get

$$\mathbf{i}: \ 1.667\omega_2^2 = 594.6 \tag{7}$$
$$\mathbf{j}: \ 1.667\alpha_2 = -170.1 \tag{8}$$

Solving for ω_2 (note that we get a $\pm$ sign on ω_2 but the direction is given in the sketch) and α_2, and substituting in (4) and (5), the angular velocity and angular acceleration vectors of link 2 are

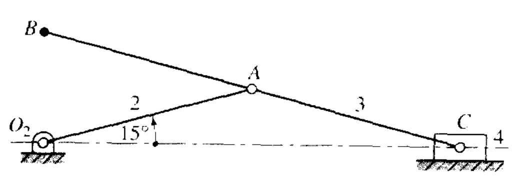
$$\boxed{\boldsymbol{\omega}_2 = -18.89\,\mathbf{k} \text{ rad/s}} \tag{9}$$
$$\boxed{\boldsymbol{\alpha}_2 = -102\,\mathbf{k} \text{ rad/s}^2} \tag{10}$$

Since we now know $\boldsymbol{\omega}_2$ and $\boldsymbol{\alpha}_2$, the acceleration of C can be found from either point A or B; I'll use A:

$$\boxed{\begin{aligned}\mathbf{a}_C &= \mathbf{a}_A + \boldsymbol{\omega}_2 \times (\boldsymbol{\omega}_2 \times \mathbf{r}_{CA}) + \boldsymbol{\alpha}_2 \times \mathbf{r}_{CA} \\ &= 2.64\,\mathbf{i} + 434.6\,\mathbf{j} \text{ ft/s}^2\end{aligned}} \tag{11}$$

BTW, this problem cannot be solved in ADAMS—not enough information is given.

Problem 4.8: (Analytical Solution) The problem is shown below:



Here $\omega_2 = 20$ rad/s ccw and $\alpha_2 = 140$ rad/s^2 ccw (link 2 is moving up and speeding up). We wish to find the velocity and acceleration of B and the angular acceleration of link 3.

Using the "Two Pts. on a Body" equation, you can relate the motion of A to O_2, then A to C (not to B; we know the path of C). Oops, I guess we "know" the path of B is vertical, but it's not as obvious.

First a velocity analysis, then acceleration.

(a) Velocity: The velocity of A can be related to points O_2 using the "2 points on a body" equation:

$$\mathbf{v}_A = \boldsymbol{\omega}_2 \times \mathbf{r}_{AO_2} = -0.5176\mathbf{i} + 1.9319\mathbf{j} \text{ m/s} \tag{12}$$

Also, by inspection $\omega_3 = 20$ rad/s cw ($\boldsymbol{\omega}_3 = -20\mathbf{k}$), so we can relate the velocity of points A and B:

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_3 \times \mathbf{r}_{BA} = 3.8637\mathbf{j} \text{ m/s}, \tag{13}$$

so point B is moving perfectly upwards at 3.86 m/s (B describes a straight line after all).

(b) Acceleration: Same kind of approach; can use "2 points on a body" throughout.

$$\mathbf{a}_A = \boldsymbol{\alpha}_2 \times \mathbf{r}_{AO_2} + \boldsymbol{\omega}_2 \times (\boldsymbol{\omega}_2 \times \mathbf{r}_{AO_2}) = -42.26\mathbf{i} + 3.17\mathbf{j} \text{ m/s}^2 \tag{14}$$

We know the direction of $\mathbf{a}_C$; it's in the horizontal ($\mathbf{i}$) direction, so relate the acceleration of C to A:

$$\mathbf{a}_C = \mathbf{a}_A + \boldsymbol{\alpha}_3 \times \mathbf{r}_{CA} + \boldsymbol{\omega}_3 \times (\boldsymbol{\omega}_3 \times \mathbf{r}_{CA}) \tag{15}$$

Substituting numerical values,

$$a_C\mathbf{i} = -80.897\mathbf{i} + 13.523\mathbf{j} + 0.0259\alpha_3\mathbf{i} + 0.0966\alpha_3\mathbf{j} \text{ m/s}^2 \tag{16}$$

Separating the $\mathbf{i}$ and $\mathbf{j}$ components of (16) and solving, you get:

$$\boxed{\boldsymbol{\alpha}_3 = -140\mathbf{k} \text{ rad/s}^2 \text{ (cw)}} \tag{17}$$

Finally, relate $\mathbf{a}_B$ to $\mathbf{a}_A$ (you could also use $\mathbf{a}_C$), and get

$$\boxed{\begin{aligned}\mathbf{a}_B &= \mathbf{a}_A + \boldsymbol{\alpha}_3 \times \mathbf{r}_{BA} + \boldsymbol{\omega}_3 \times (\boldsymbol{\omega}_3 \times \mathbf{r}_{BA}) \\ &= 6.3403\,\mathbf{j} \text{ m/s}^2 \text{ (upwards)}\end{aligned}} \tag{18}$$

(ADAMS Solution) Since ADAMS is a CAD package designed to simulate the behavior of a mechanism over a selected time period, I wanted you to solve for the parameters that would allow the mechanism to start from rest and accelerate such that at $\theta_2 = 15°$ the mechanism would have exactly the conditions stated in the text.

The boundary conditions (B.C.) are—at $\theta_2 = 15°$:

$$\omega_2 = \dot{\theta}_2 = 20 \text{ rad/s} \tag{19}$$
$$\alpha_2 = \ddot{\theta}_2 = 140 \text{ rad/s}^2 \tag{20}$$

The condition that mechanism start from rest is an initial condition (I.C.) for $\theta_2(0)$, and is

$$\omega_2(0) = 20 \text{ rad/s} \tag{21}$$

It turns out that this is insoluble without a knowledge of the time-dependent behavior of acceleration $\ddot{\theta}_2(t)$. So I proposed that the acceleration of θ_2 be constant, thus

$$\ddot{\theta}_2(t) = 140 \text{ rad/s}^2 \tag{22}$$

Given (22), we can now solve for the time-dependent behavior of ω_2 and θ_2:

$$\omega_2(t) = \dot{\theta}_2(t) = \int \ddot{\theta}_2(t)dt = \int (140)\,dt = 140t + C_1 \tag{23}$$
$$\theta_2(t) = \int \dot{\theta}_2(t)\,dt = \int (140t + C_1)\,dt = 70t^2 + C_1t + C_2 \tag{24}$$

Elimination of Constants of Integration: First apply I.C. (21) to (23):

$$\omega_2(0) = 140(0) + C_1 = 20 \implies C_1 = 0. \tag{25}$$

So equations (23) and (24) now simplify to

$$\omega_2(t) = 140t \text{ rad/s} \tag{26}$$

$$\theta_2(t) = 70t^2 + C_2 \text{ rad} \tag{27}$$

Now—one cannot apply the velocity B.C. of (19) directly. Instead, you must solve (26) for the *time* (call it t_1) when $\omega_2 = 20$ rad/s, then substitute that *time* into (27) to solve for constant C_2.

$$\omega_2(t_1) = 140t_1 = 20 \implies t_1 = \frac{20}{140} = \frac{1}{7} \text{ sec} \tag{28}$$

So it is at $t_1 = \frac{1}{7}$ sec when we want $\theta_2 = 15° = \frac{15\pi}{180} = \frac{\pi}{12}$ rad. Substituting these conditions into (27) we get

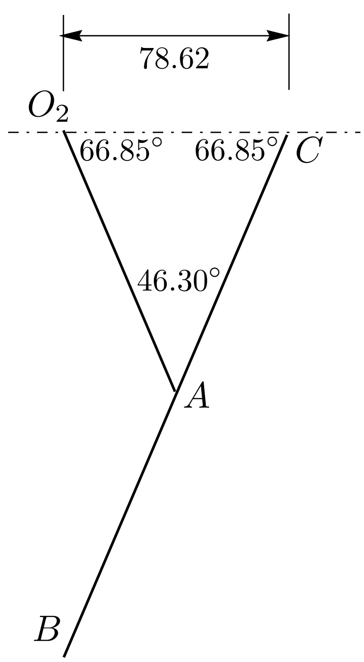
$$\theta_2(t_1) = 70t_1^2 + C_2 = 70\left(\frac{1}{7}\right)^2 + C_2 = \frac{\pi}{12} \tag{29}$$

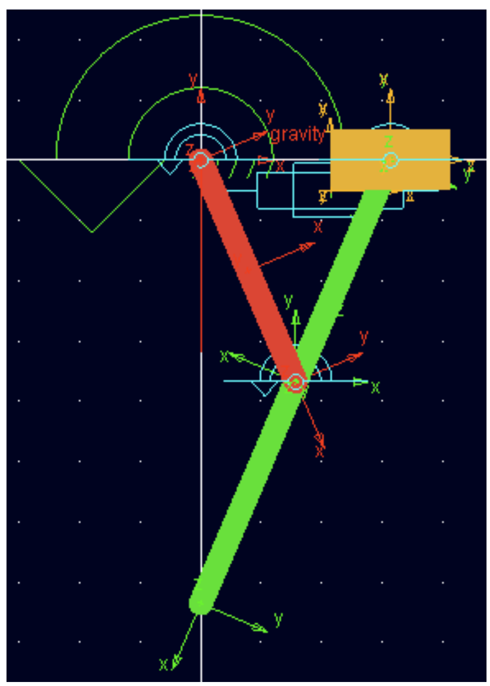
and now solve for C_2:

$$C_2 = \frac{\pi}{12} - \frac{70}{49} = -1.1668 \text{ rad} = -66.85° \tag{30}$$

Referring to (27), it is clear that C_2 is the **initial angle** of link 2. So when you build the ADAMS model, you must build it (there may be another way, but I don't know it) with link 2 initially at an angle of −66.85°.

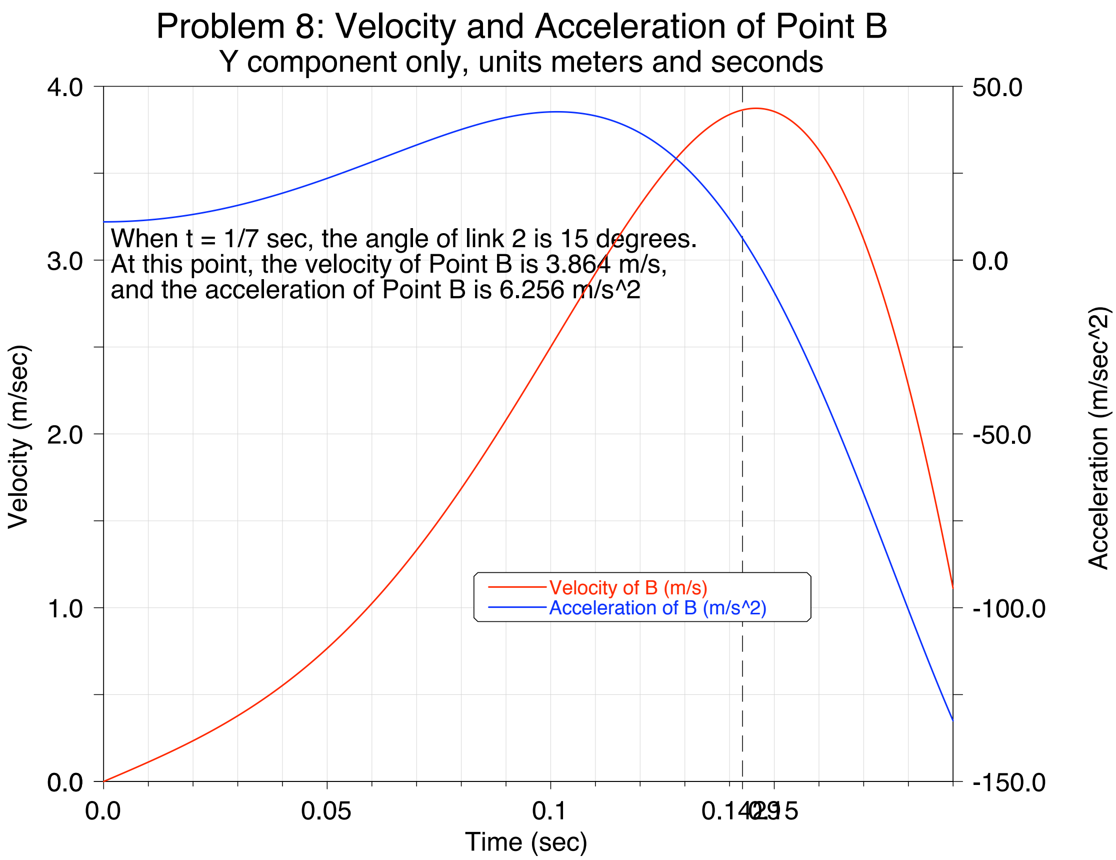
So, unfortunately, you need to solve the geometry of the mechanism at this initial angle. Refer to the figure at left below.

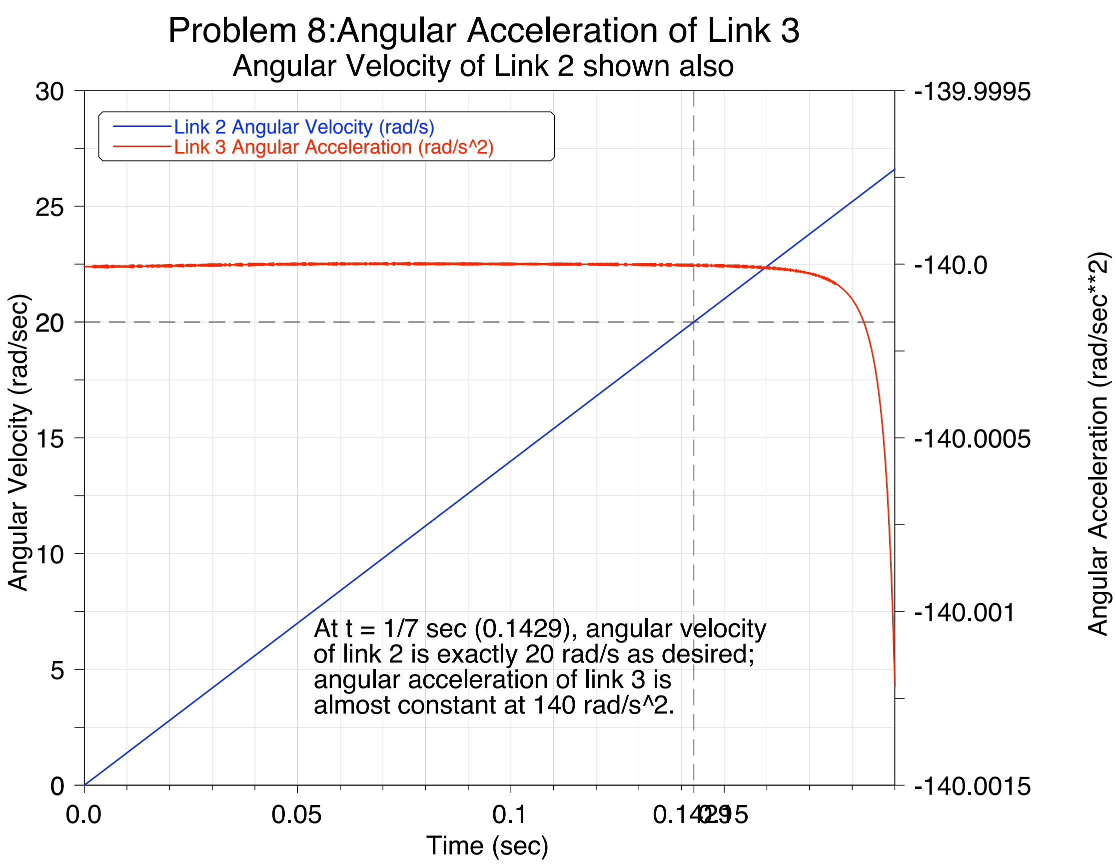




This figure has all the dimensions needed to create the ADAMS model, a screenshot of which is shown at right above. Note that the initial configuration of the ADAMS model agrees with the sketch.

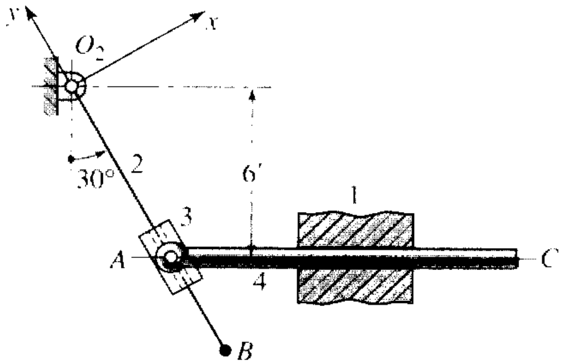
The plots of the desired kinematical quantities are on the next page.





Pretty good agreement with the analytical method.

Problem 4.32: The *Rapson's Slide* is shown below, where the velocity of actuating rod AC is 10 ft/min to the left (since no acceleration is given, assume it's zero). We are to find the angular acceleration of the tiller (link 2).



(Analytical Solution) This problem cannot be done with just the "2 points on a body" equation; there is sliding motion between bodies 2 and 3, and body 3 is rotating. Point A is attached to body 3, not body 2. Note that the xyz coordinate system is angled. As I suggested, I will use units of "ft, rad, min" for linear, angular, and time measure in my result.

We need to do a velocity analysis first (as usual), then acceleration.

KNOWN PATH: We know the path of point A relative to body 2 — it's a straight line in the y direction. This will dictate how we approach the analysis.

(a) Velocity: Since we know the path of A relative to 2, write the "One point moving on a body" equation for the velocity of A:

$$\mathbf{v}_A = \mathbf{v}_{A_2} + {}^2\mathbf{v}_A \tag{31}$$

where we know that point A has a velocity of 10 ft/min to the left, point A_2 is fixed to link 2 and follows a circular path around O_2, the motion of A relative to 2 is along O_2B. Expressing these velocities analytically, we have

$$\mathbf{v}_A = 10(-\cos 30°\mathbf{i} + \sin 30°\mathbf{j}) = -8.66\mathbf{i} + 5\mathbf{j} \text{ ft/min} \tag{32}$$

$$\mathbf{v}_{A_2} = \boldsymbol{\omega}_2 \times \mathbf{r}_{AO_2} = \omega_2\mathbf{k} \times (-6.928\mathbf{j}) = 6.928\omega_2\mathbf{i} \text{ ft/min} \tag{33}$$

$${}^2\mathbf{v}_A = {}^2v_A\,\mathbf{j} \tag{34}$$

Equating (30) and (31) for $\mathbf{v}_{A_3}$ and substituting values, I got

$$\omega_2 = -1.25 \text{ rad/min (cw direction)} \tag{35}$$

$${}^2v_A = 5 \text{ ft/min (will need this for Coriolis acceleration term)} \tag{36}$$

(b) Acceleration: Again, assume $\mathbf{a}_A = 0$. The relative acceleration equation is

$$\mathbf{a}_A = \mathbf{a}_{A_2} + {}^2\mathbf{a}_A + 2\boldsymbol{\omega}_2 \times {}^2\mathbf{v}_A \tag{37}$$

where the same motion information applies here as in the velocity analysis.

$$\mathbf{a}_A = \boldsymbol{\alpha}_2 \times \mathbf{r}_{AO_2} + \boldsymbol{\omega}_2 \times (\boldsymbol{\omega}_2 \times \mathbf{r}_{AO_2}) = 6.928\alpha_2\mathbf{i} + 10.825\mathbf{j} \text{ ft/min}^2 \tag{38}$$

$${}^2\mathbf{a}_A = {}^2a_A\,\mathbf{j} \tag{39}$$

$$2\boldsymbol{\omega}_2 \times {}^2\mathbf{v}_A = 12.5\mathbf{i} \text{ ft/min}^2 \tag{40}$$

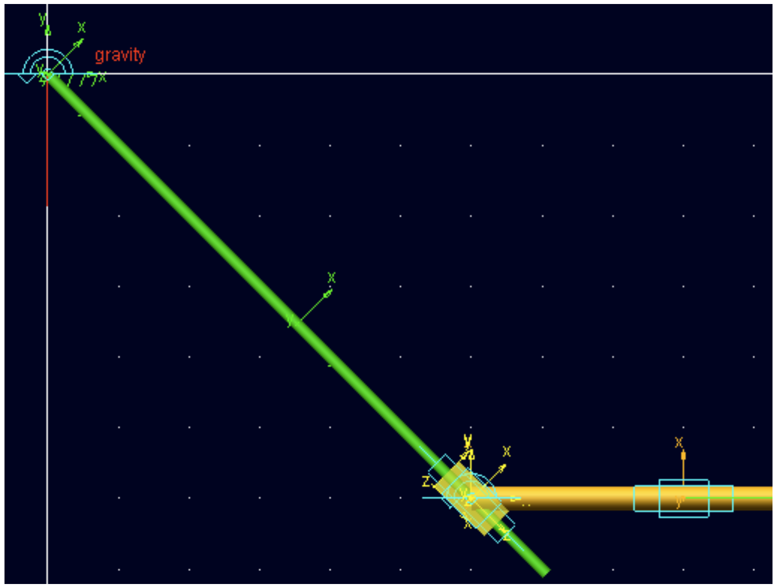
Since $\mathbf{a}_A = 0$, use (37–39) to solve this mechanism; I found that

$$\boxed{\boldsymbol{\alpha}_2 = -1.804\mathbf{k} \text{ rad/min}^2 \text{ (cw direction)}} \tag{41}$$

Although not requested, I found that ${}^2a_A = -10.8253$ ft/min^2 (in $-y$ direction...slider is slowing down on the rod).

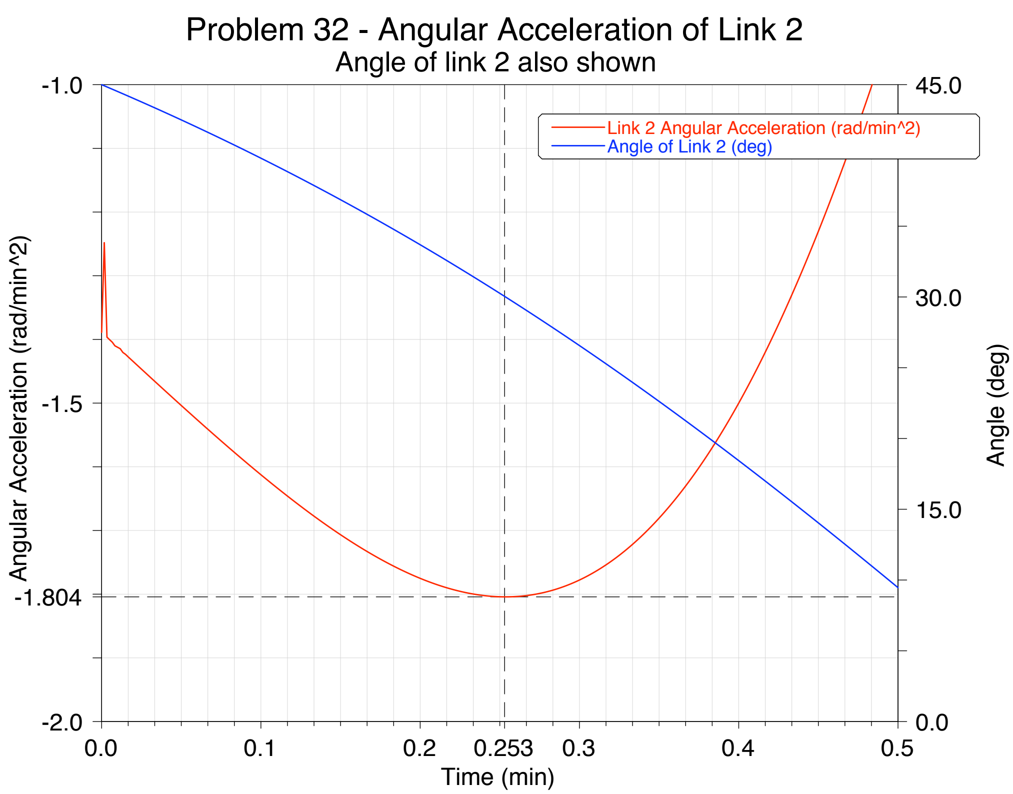
Problem 4.32 (con't.) **(ADAMS Solution)** Like Problem 8, I constructed the mechanism at an initial configuration so I could simulate it for a time duration and observe the behavior as it passes through the desired point.

I started it at an angle of 45°, which allows some motion to occur before it reaches 30°. A screenshot at the starting point is



There is a revolute joint between link 2 (tiller) and ground at the upper left, a revolute joint between link 4 and the slider (link 3) at point A, a translational joint between link 4 and ground at lower right, and a translational joint between link 3 (slider) and link 2 (tiller).

The plot of the angular acceleration of link 2 is shown below. I also plotted the angle of link 2; the time (0.253 min) when the angle of link 2 is 30° is shown. I also drew a line at the value of α_2 at that time; the value of -1.804 rad/min^2 agrees with the analytical result.



It is interesting that the time at which link 2 is at 30° is also when its angular acceleration is a minimum.