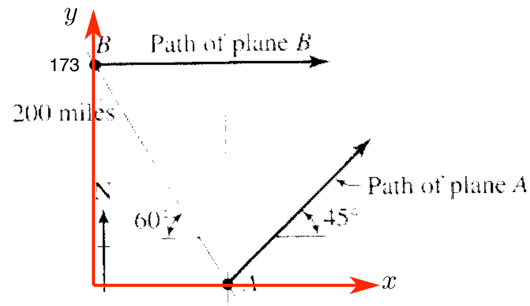
Chapter 3 HW Solution

Problem 3.6: I placed an xy coordinate system at a convenient point (origin doesn't really matter).



The positions of both planes are given by

$$\mathbf{r}_B = v_B t\mathbf{i} + 173\mathbf{j} \text{ mi} \tag{1}$$

$$\mathbf{r}_A = 100\mathbf{i} + v_A t(0.707\mathbf{i} + 0.707\mathbf{j}) \text{ mi} \tag{2}$$

The distance between A and B is the length (magnitude) of vector $\mathbf{r}_{AB}$, which is the vector from B to A (or the reverse; distance is the same). So we have

$$\mathbf{r}_{BA} = \mathbf{r}_B - \mathbf{r}_A \tag{3}$$

However, we're really interested in the *magnitude* of the vector $\mathbf{r}_{BA}$, denote that magnitude by length l:

$$l(t) = |\mathbf{r}_{BA}| = |(74t - 100)\mathbf{i} + (-276t + 173)\mathbf{j}| \text{ mi} \tag{4}$$

The magnitude (Euclidean norm) of a vector is simply the square root of the sum of the components squared, so

$$l(t) = \sqrt{(74t - 100)^2 + (-276t + 173)^2} \text{ mi} \tag{5}$$

Parts (a) and (b) of the problem should have been reversed: first you have to find the time, then you can find the distance. This is just a "standard" function minimization problem; *e.g.* take the first derivative and set it equal to zero.

(b) The differentiation is simpler if you realize that when $l(t)$ is at a minimum, so is $[l(t)]^2$. This removes the square root, and

$$\frac{d\,[l(t)]^2}{dt} = \frac{d}{dt}\left[81,652t^2 - 110,296t + 39,929\right] = 163,304t - 110,296 = 0 \tag{6}$$

Solving (13) yields the *time* when the minimum separation occurs, which is

$$t = \frac{110,296}{163,304} = 0.675 \text{ hr} = 40.52 \text{ min} \tag{7}$$

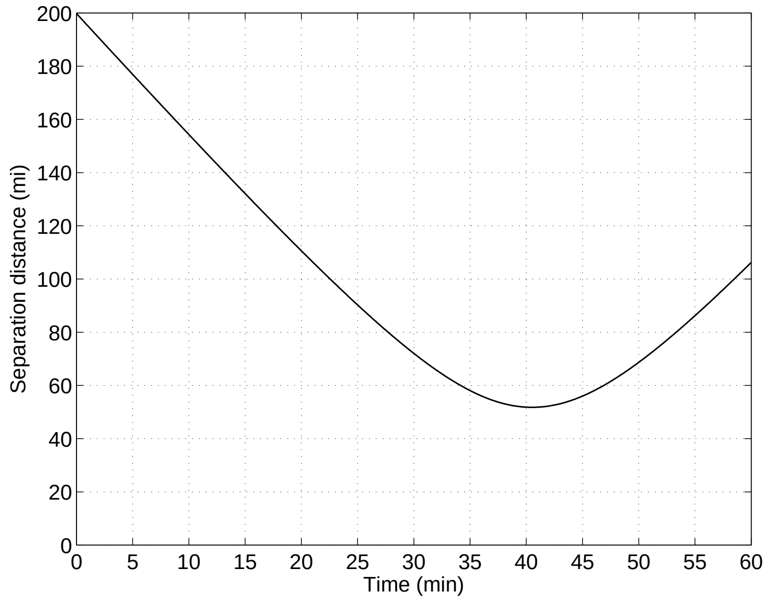
If the planes leave at 6:00 p.m., then the time of minimum separation is

$$\boxed{t_{min} = 6:40:31 \text{ p.m.}} \tag{8}$$

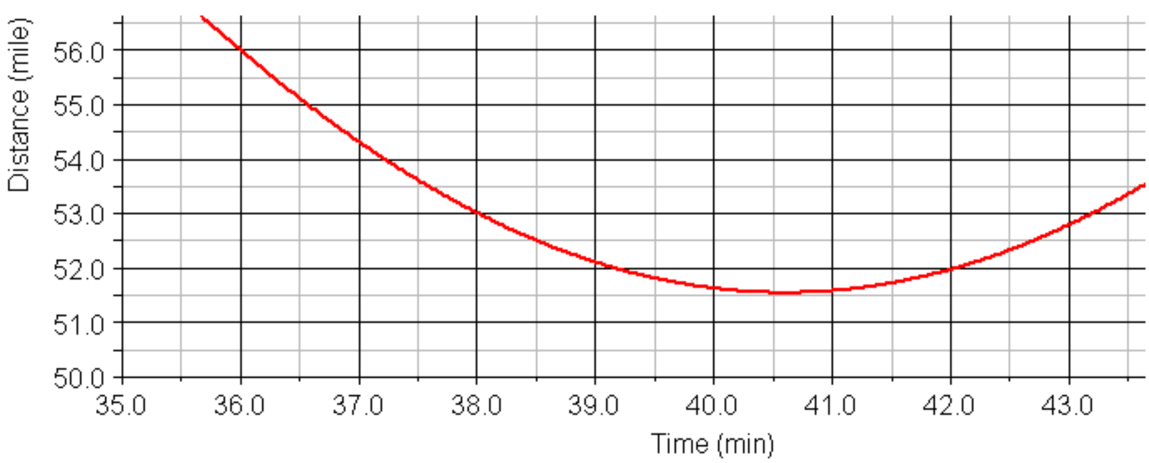
(a) To find the actual separation distance, substitute t_{min} (expressed in fractional hours of (14)) into equation (12). The result I found was

$$\boxed{l_{min} = 51.79 \text{ mi}} \tag{9}$$

Although not required, I couldn't resist plotting separation distance *vs* time (MATLAB plot on next page); it seems to agree with my result.

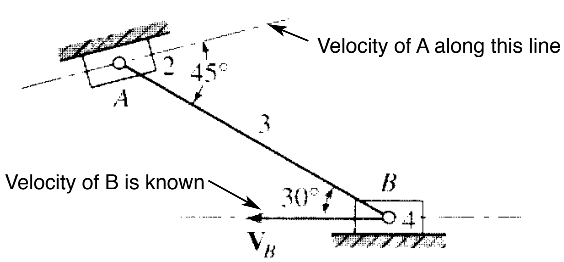


(c) Although not required, you can *also* do this problem with ADAMS, you have to set up two bodies with the velocities of the planes, and a "`Point-to-Point`" measure for the distance between the two planes. You get the following plot:



The minimum of the ADAMS plot occurs at $t = 40.6$ minutes, which is pretty close to the previous result. The ADAMS separation distance is 51.5445 miles; again pretty close.

Problem 3.8: Do this analytically.



Points A and B are both on link 3, so they're related by the "2 pts on a body" equation:

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega}_3 \times \mathbf{r}_{BA} \tag{10}$$

Velocity $\mathbf{v}_B$ is known (along lower plane), the *direction* of velocity $\mathbf{v}_A$ is known (along upper plane), and the angular velocity $\boldsymbol{\omega}_3$ is in the k direction (perpendicular to the plane). From the angles given, the upper plane is at angle of

15° from the horizontal. From inspection, block A is moving to the left, so we have

$$v_A(-\cos 15°\mathbf{i} - \sin 15°\mathbf{j}) = -40\mathbf{i} + \omega_3\mathbf{k} \times 0.4(-\cos 30°\mathbf{i} + \sin 30°\mathbf{j}) \tag{11}$$

Separating the **i** and **j** equations, there are

$$\mathbf{i} : -0.9659v_a = -40 - 0.2\omega_3 \tag{12}$$
$$\mathbf{j} : -0.2588v_a = -0.3463\omega_3 \tag{13}$$

In matrix form, equations (12)–(13) are

$$\begin{bmatrix} -0.9659 & 0.2 \\ -0.2588 & 0.3463 \end{bmatrix} \begin{bmatrix} v_a \\ \omega_3 \end{bmatrix} = \begin{bmatrix} -40 \\ 0 \end{bmatrix} \tag{14}$$

Solving, we get

$$\begin{bmatrix} v_A \\ \omega_3 \end{bmatrix} = \begin{bmatrix} 48.9935 \text{ m/s} \\ 36.6143 \text{ rad/s} \end{bmatrix} \tag{15}$$

In terms of vectors, we have $\mathbf{v}_A = v_A(-\cos 15°\mathbf{i} - \sin 15°\mathbf{j})$, so

$$\boxed{\begin{aligned} &\mathbf{v}_A = -47.3241\mathbf{i} - 12.6805\mathbf{j} \text{ m/s} && (16) \\ &\boldsymbol{\omega}_3 = 36.6143\mathbf{k} \text{ rad/s} && (17) \end{aligned}}$$

So link 3 is rotating **CCW**, which I think agrees with the sketch. And block A is sliding a little faster than block B (49 m/s compared with 40 m/s).

Problem 3.9 In the 4-bar mechanism shown below, link 2 is driven at a constant angular velocity of $\omega_2 = 45$ rad/s CCW. We want to find the angular velocities $\boldsymbol{\omega}_3$ and $\boldsymbol{\omega}_4$.

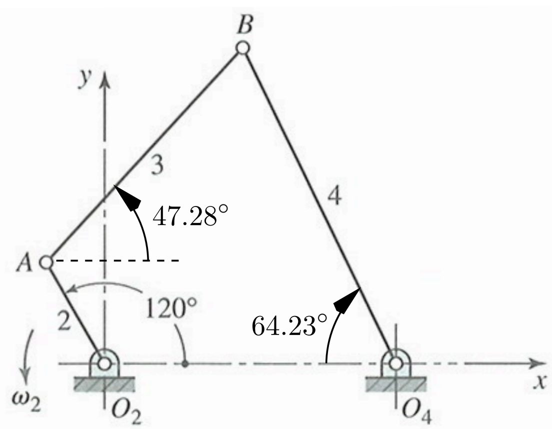


Figure P3.9 $R_{AO_2} = 4$ in, $R_{BA} = 10$ in, $R_{O_4O_2} = 10$ in, $R_{BO_4} = 12$ in.

You will need the angles I found above in the analysis. You are to do this problem both analytically and using ADAMS.

(a) *Analytical Solution.* This can be done using only "2 point on a body" throughout. Start by finding the velocity of A:

$$\mathbf{v}_A = \underbrace{\mathbf{v}_{O_2}}_{=0} + \boldsymbol{\omega}_2 \times \mathbf{r}_{O_2A} = 45\mathbf{k} \times (-2\mathbf{i} + 3.46\mathbf{j}) = -155.9\mathbf{i} - 90\mathbf{j} \text{ in/s} \tag{18}$$

Next relate the velocities of A and B:

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_3 \times \mathbf{r}_{AB} \tag{19}$$

where $\boldsymbol{\omega}_3 = \omega_3 \mathbf{k}$ rad/s and $\mathbf{r}_{AB} = 6.78\mathbf{i} + 7.35\mathbf{j}$ in. Substituting for $\mathbf{v}_A$ and evaluating, we get

$$\mathbf{v}_B = (-155.9 - 7.35\omega_3)\mathbf{i} + (-90 + 6.78\omega_3)\mathbf{j} \text{ in/s} \tag{20}$$

Now relate the velocities of B and O_4:

$$\mathbf{v}_B = \underbrace{\mathbf{v}_{O_4}}_{=0} + \boldsymbol{\omega}_4 \times \mathbf{r}_{BO_4} \tag{21}$$

where $\boldsymbol{\omega}_4 = \omega_4 \mathbf{k}$ rad/s and $\mathbf{r}_{BA} = -5.22\mathbf{i} + 10.81\mathbf{j}$ in. Evaluting this, we get

$$\mathbf{v}_B = -10.81\omega_4 \mathbf{i} - 5.22\omega_4 \mathbf{j} \text{ in/s} \tag{22}$$

Equate (20) and (22) to obtain

$$\mathbf{i} : -155.9 - 7.35\omega_3 = -10.81\omega_4 \tag{23}$$
$$\mathbf{j} : -90 + 6.78\omega_3 = -5.22\omega_4 \tag{24}$$

I like to express these in matrix form:

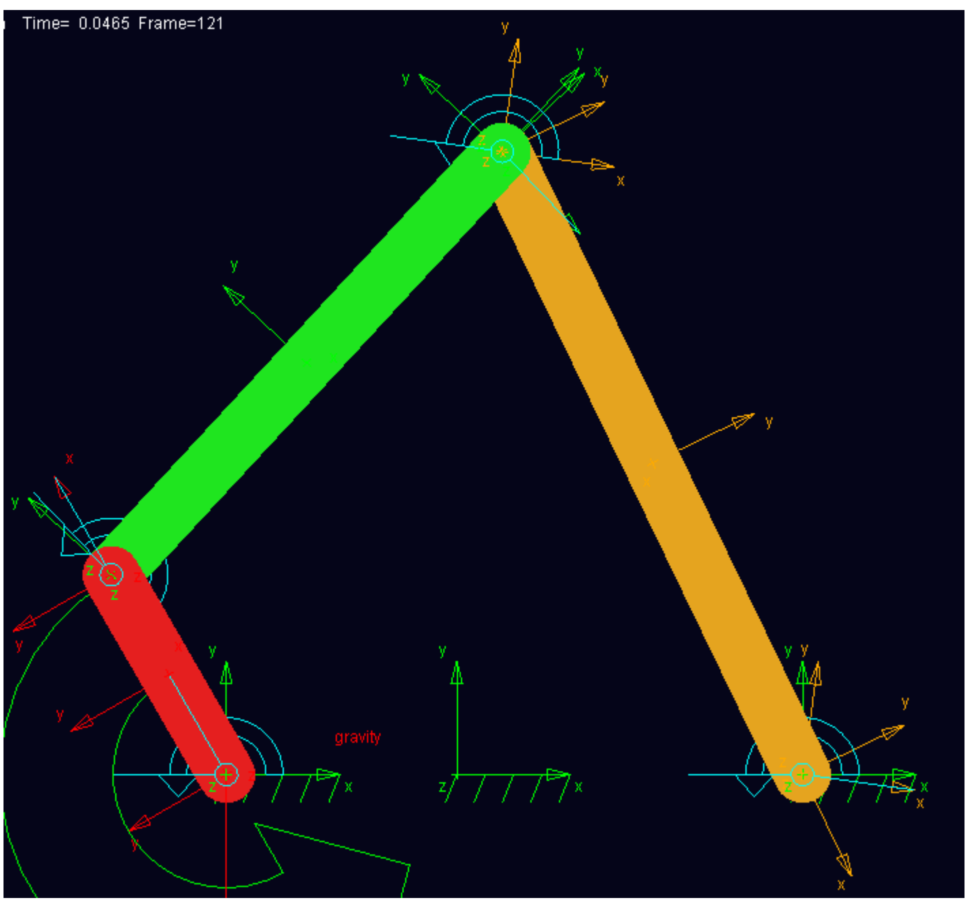
$$\begin{bmatrix} -7.35 & 10.81 \\ 6.78 & 5.22 \end{bmatrix} \begin{bmatrix} \omega_3 \\ \omega_4 \end{bmatrix} = \begin{bmatrix} 155.9 \\ 90 \end{bmatrix} \tag{25}$$

I solved these with MATLAB to yield

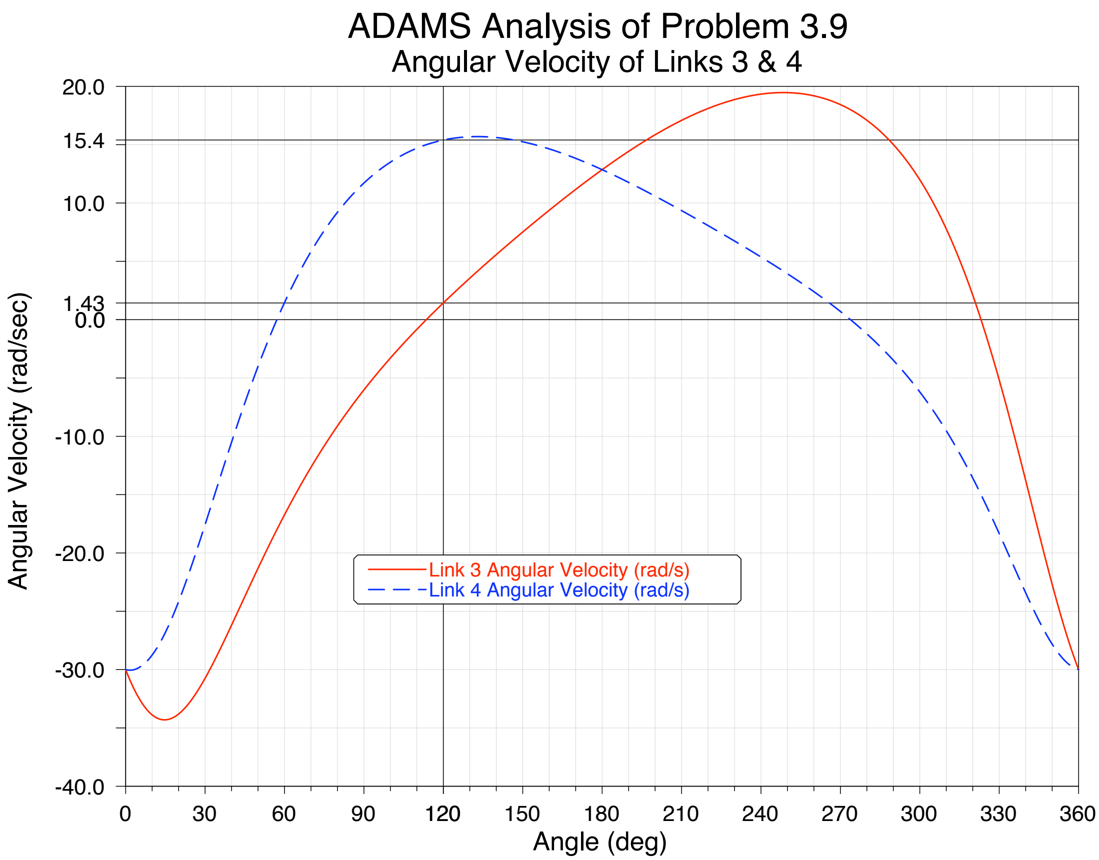
$$\boxed{\begin{aligned} \boldsymbol{\omega}_3 &= 1.42\mathbf{k} \text{ rad/s} \\ \boldsymbol{\omega}_4 &= 15.39\mathbf{k} \text{ rad/s} \end{aligned}} \tag{26, 27}$$

So both angular velocities are CCW, and link 4 is much faster than link 3. I guess that looks okay.

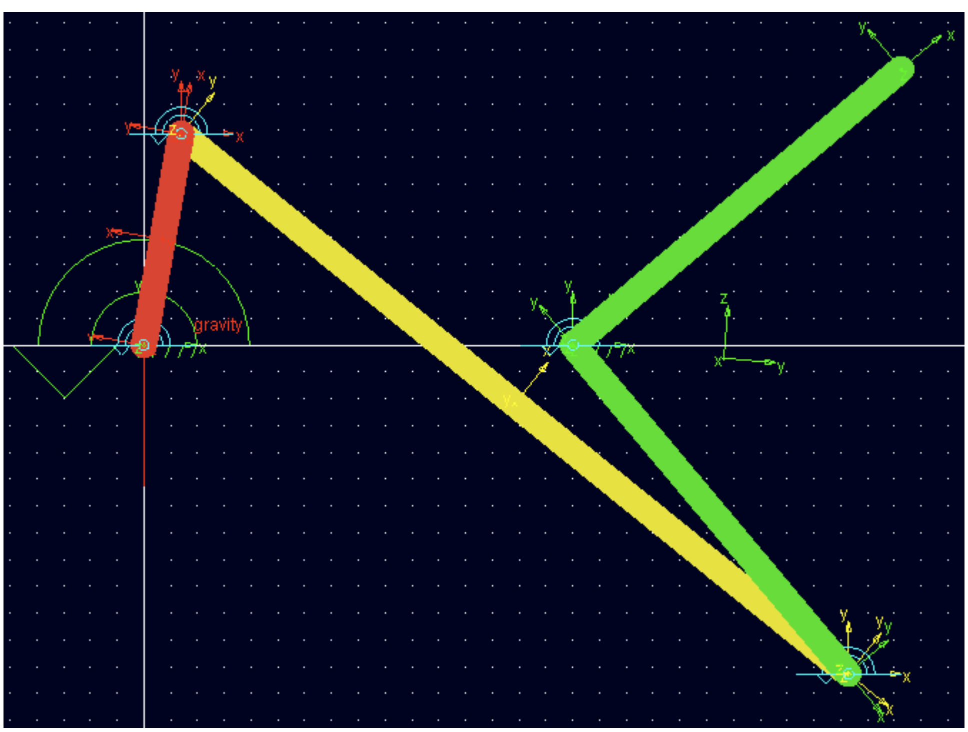
(b) ADAMS*Solution.* An ADAMS screenshot of the mechanism (at $\theta_2 = 120°$) is shown below. The velocity plot is shown on the next page.



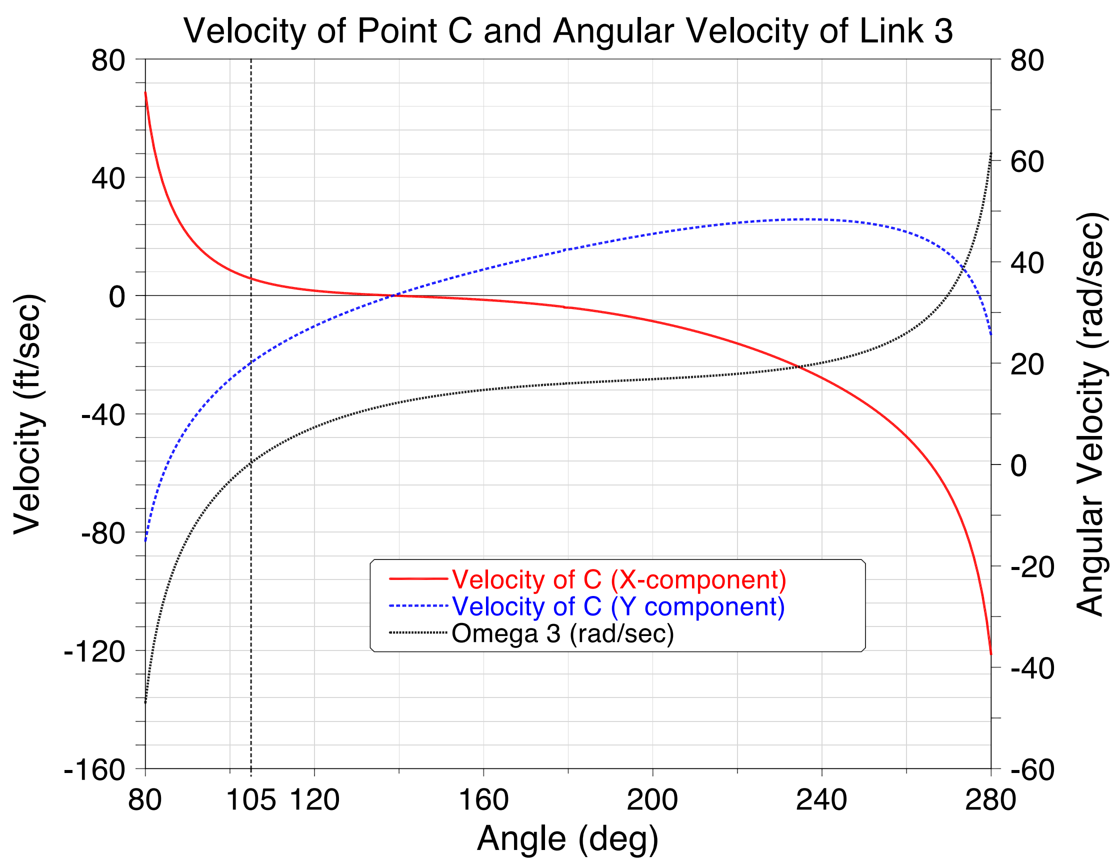
Here's the velocity plot, with lines drawn at 120°. The results at the angle agree with the analytical.



Problem 3.11 (ADAMS Only). A screenshot of my linkage in the initial position is shown below:



The velocity plot for this problem is shown below. Note that the velocity of point C is quite large near the limits of motion (typical).

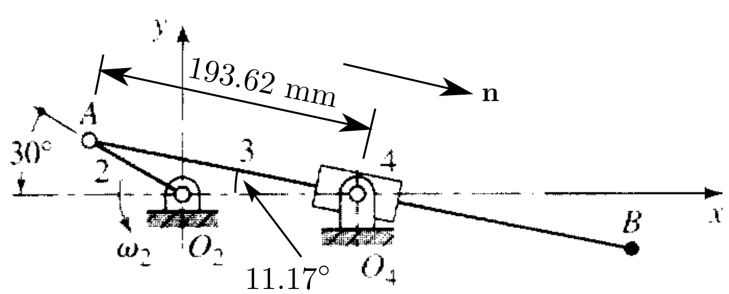


Problem 3.15: A position analysis using the loop closure equation shows that

$$r_{AO_4} = 193.62 \text{ mm} \tag{28}$$

$$\text{Angle of } AB \text{ with horizontal} = 11.1677°, \tag{29}$$

and both these values will be needed. The figure is shown below, with those numerical values.



(a) *Analytical Velocity:* For the analytical velocity analysis, you'll need to use both the "2 points on a body" and the "one point moving on a body" equations.

Find the velocity of A using points O_2 (stationary) and A and the "two points on a body" relationship:

$$\mathbf{v}_A = \boldsymbol{\omega}_2 \times \mathbf{r}_{O_2A} = -2250\mathbf{i} - 3897\mathbf{j} \text{ mm/s} \tag{30}$$

So the velocity of A is *known.*

Next find the velocity of A again, but now you relate links 3 and 4. You know the path of A relative to body 4. For this situation use the "one point moving on a body" equation, with A as the point, and 4 as the body, therefore written as follows:

$$\mathbf{v}_A = \mathbf{v}_{A_4} + {}^4\mathbf{v}_A \tag{31}$$

Consider equation (31) *very carefully!!* Point A_4 is point "A" in the figure. However...point A_4 is a point that is coincident with A, but *FIXED TO BODY 4*. You may think of it as a hypothetical extension of body 4 up to point A. The path of A_4 is a circular arc centered at O_4. Therefore, the velocity $\mathbf{v}_{A_4}$ is *tangential* to that circle, and hence perpendicular to AB. So we know the *direction* of $\mathbf{v}_{A_4}$, but not its magnitude.

Finally, term ${}^4\mathbf{v}_A$ is the velocity of A relative to body 4. I visualize this by mentally "fixing" body 4, then examining the motion of A.

All right, let's solve the problem. Referring to equation (31), we know velocity $\mathbf{v}_A$, it's given in equation (30). Next express $\mathbf{v}_{A_4}$ as an unknown magnitude in a known direction. This can either be done using v_{A_4} multiplied by the direction of the velocity, or using $\boldsymbol{\omega}_4$ and the cross product. Since the problem statement asks for the angular velocities of 3 and 4 (they're equal), I'll do that:

$$\mathbf{v}_{A_4} = \boldsymbol{\omega}_4 \times \mathbf{r}_{O_4A} = \omega_4\mathbf{k} \times (-189.95\mathbf{i} + 37.51\mathbf{j}) = -37.51\omega_4\mathbf{i} - 189.95\omega_4\mathbf{j} \tag{32}$$

Now express ${}^4\mathbf{v}_A$ as an unknown magnitude in a known direction:

$${}^4\mathbf{v}_A = {}^4v_A \underbrace{(\cos(11.17°\mathbf{i} - \sin(11.17°\mathbf{j})}_{\text{along } AB} = {}^4v_A(0.9811\mathbf{i} - 0.1937\mathbf{j}) \tag{33}$$

Substituting into (31) and separating the $\mathbf{i}$ and $\mathbf{j}$ components, we get

$$\mathbf{i}: \quad -37.5\omega_4 + 0.9811v_{A_3/4} = -2250 \tag{34}$$
$$\mathbf{j}: \quad -189.95\omega_4 - 0.1937v_{A_3/4} = -3897 \tag{35}$$

Angular velocities: Solving (34) and (35), we get results

$$\omega_4 = \omega_3 = 22 \text{ rad/s} \tag{36}$$
$$v_{A_3/4} = -1453 \text{ mm/s} \tag{37}$$

So the angular velocity vector of links 3 and 4 is

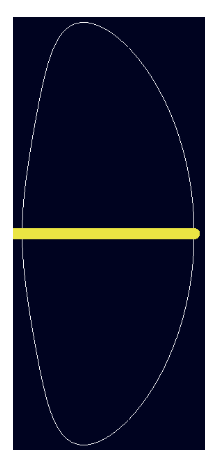
$$\boxed{\boldsymbol{\omega}_3 = \boldsymbol{\omega}_4 = 22\,\mathbf{k} \text{ rad/s (CCW)}} \tag{38}$$

Velocity of point B: Knowing ω_3 we can relate the velocity of B to the velocity of A:

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_3 \times \mathbf{r}_{AB} = \mathbf{v}_A + 22\mathbf{k} \times (392.42\mathbf{i} - 77.49\mathbf{j}) \tag{39}$$
$$= -545.3\mathbf{i} + 4736.3\mathbf{j} \text{ mm/s} \tag{40}$$
$$= -0.5453\mathbf{i} + 4.7363\mathbf{j} \text{ m/s} = 4.77\angle 96.56° \text{ m/s} \tag{41}$$

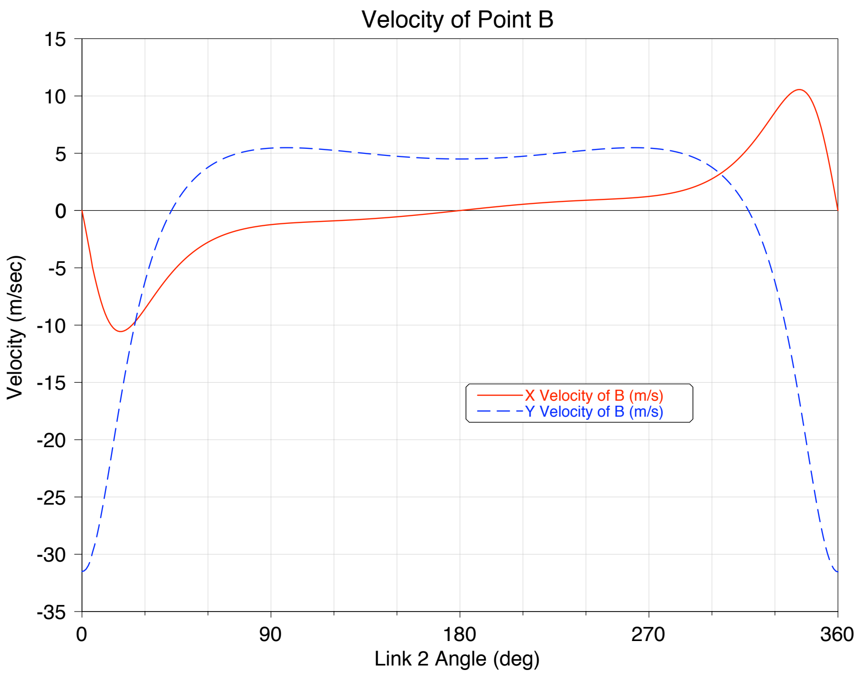
I expressed the last result for $\mathbf{v}_B$ in "polar" form; this may be easier to visualize.

(b) ADAMS *Analysis.* The path of Point B is shown at right. The yellow "bar" across the center is simply the initial position of link 3. What is *NOT* shown in this plot is the speed (magnitude of velocity) of point B as it moves along the path.



In particular, the y velocity is quite large as θ_2 is near zero. Hopefully this will be shown in the velocity plots on the next page.

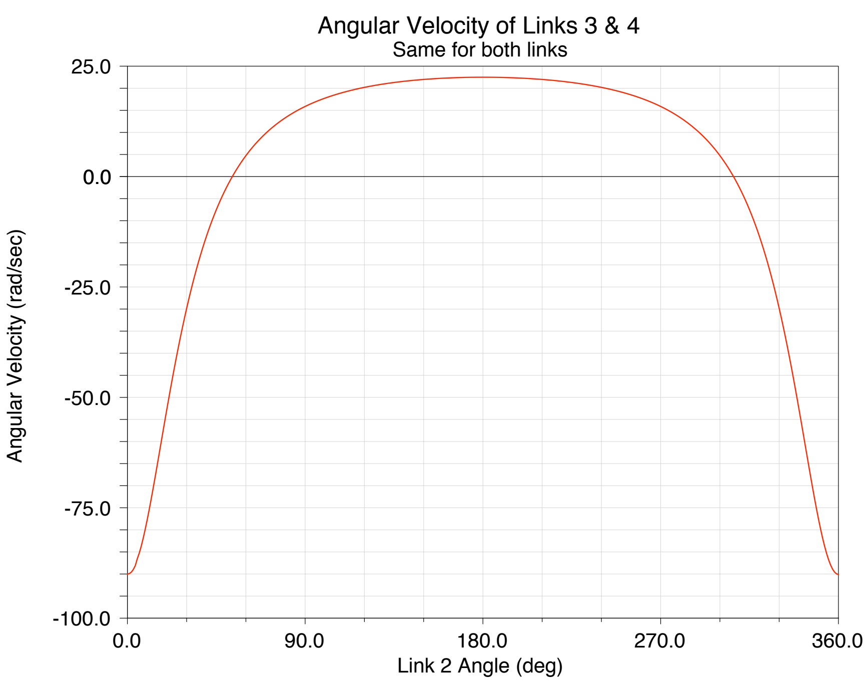
The plot of the velocity of point B appears below; the y velocity is large near $\theta_2 = 0°$.



At $\theta_2 = 150°$ the values for the velocity components are

$$\boxed{\begin{aligned}(v_B)_x &= -0.546 \text{ m/s} \quad (42)\\ (v_B)_y &= 4.7351 \text{ m/s} \quad (43)\end{aligned}}$$

which agree quite well with the analytical solution. The plot of ω_3 (same as ω_4) is:



At $\theta_2 = 150°$ the value of the angular velocity is

$$\boxed{\omega_3 = \omega_4 = 21.9979 \text{ rad/s}} \quad (44)$$

which also agrees well.