Problem 1.1. Here are nine photos of planar four-bar linkages.

Figure 1: Planar four-bar linkages in practice.

I have several photos of motorcycle engines in various stages of disassembly, showing portions of the valvetrain and crankshaft/connecting rod linkages, but it’s difficult to see the entire linkage, so I left them out.
Problem 1.2. With link lengths of

<table>
<thead>
<tr>
<th>link</th>
<th>length</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5 in</td>
</tr>
<tr>
<td>2</td>
<td>1 in</td>
</tr>
<tr>
<td>3</td>
<td>3 in</td>
</tr>
<tr>
<td>4</td>
<td>5 in</td>
</tr>
</tbody>
</table>

this mechanism satisfies Grashof’s Law, and can continuously rotate. I’ll construct four ADAMS models of this linkage, with consecutive links fixed, and animate them during class if time permits.

(a) Link 1 fixed. A screenshot of this mechanism is shown in Figure 1(a) below. Link 2 can rotate continuously, and this is a CRANK-ROCKER mechanism.

(b) Link 2 fixed. Link 2 is now the “ground,” and link 1 is the long blue link. Here link 4 (at the top) can rotate continuously, and this is a DRAG-LINK mechanism.

(c) Link 3 fixed. Link 2 can again rotate continuously, so this is another CRANK-ROCKER mechanism.

(d) Link 4 fixed. Link 2 is now the “ground,” and link 1 is the long blue link. Here link 4 (at the top) can rotate continuously, and this is a DOUBLE-ROCKER mechanism.

Figure 2: The four inversions of Problem 2.
Problem 1.4. I constructed the ADAMS model of this problem; I used two separate elements for link 3, the “merged” them. I selected a WHITE background in this case; may be easier to print. The path of point C is shown in black.

Figure 3: ADAMS model for Problem 4. Path of point C shown in black.

Problem 1.5: There are four mechanisms shown in Figure 4 (poorly-scanned figure on the next page); in each case you apply the Kutzbach criterion

\[
m = 3(n - 1) - 2j_1 - j_2
\]

in which

\begin{align*}
m &= \text{mobility (number of mechanism DOF)} \\
n &= \text{number of links} \\
j_1 &= \text{number of single DOF joints} \\
j_2 &= \text{number of two DOF joints}
\end{align*}
(a) Here $n = 6, j_1 = 7$ (7 1DOF pin joints), and $j_2 = 0$, so

$$m = 3(n - 1) - 2j_1 - j_2 = 3(6) - 2(7) - 0 = 1 \text{ DOF (I can visualize this)}$$

(b) Here $n = 8, j_1 = 10$ (10 1DOF pin joints), and $j_2 = 0$, so

$$m = 3(n - 1) - 2j_1 - j_2 = 3(7) - 2(10) - 0 = 1 \text{ DOF (I can visualize this one, too)}$$

(c) Here $n = 7, j_1 = 9$ (8 1DOF pin joints plus rolling w/o slipping is 1 DOF), and $j_2 = 0$, so

$$m = 3(n - 1) - 2j_1 - j_2 = 3(6) - 2(9) - 0 = 0 \text{ DOF (this is a little harder to see)}$$

(d) Here $n = 4, j_1 = 3$ (3 1DOF prismatic joints), and $j_2 = 2$ (2 2DOF pin-in-slot joints), so

$$m = 3(n - 1) - 2j_1 - j_2 = 3(3) - 2(3) - 2 = 1 \text{ DOF (I definitely can’t visualize this)}$$
Problem 1.16: REAL MODEL: I built this thing using cardboard and thumbtacks—see the picture below. My wife and kids loved this model—there’s something about actually making a physical model that transcends simulations. Anyway, the complete coupler curve of the Roberts mechanism is shown in the picture—I traced it using a pen stuck through point $P$. Note that the segment between points $A$ and $B$ is quite straight.

By the way, from Grashof’s Law, $1.25 + 2.5 \leq 2.5 + 2.5$ so the coupler should have continuous rotation, and it does (if you maneuver the thumbtacks out of the way).

ADAMS MODEL (not required): I also created an ADAMS model of this mechanism and plotted the path of point $P$:

They both look pretty close. But with the ADAMS model one can really examine the “straight-line” portion...
**ADAMS plot of path of point** $P$ (not req’d). The plot below shows the same path, but plotted from within the ADAMS Postprocessor. It’s the same path, although since the axes don’t have equal scaling it is a little “squeezed.” Between $-1.25 < X < 1.25$ inches there appears to be virtually no $Y$ motion (this is the straight line).

![Path of Roberts mechanism point $P$ from ADAMS Postprocessor.](image)

**Expanded plot of $Y$ motion within straight line** (not req’d.) Below is an expanded plot of one half the motion of point $P$ between the center and the endpoint:

![Closeup of the straight-line segment showing 0.0028 inch $Y$ error.](image)

Inspection of the plot shows a maximum deviation of 0.0028 inches—less than three-thousandths of an inch. Pretty good straight line! Note the benefit of using ADAMS to simulate this mechanism.
Problem 1.17: Assume that both threads are right-handed. A thread pitch of 16NF means 16 threads/inch, and 18NF is 18 threads/inch (the major diameters 3/4 and 5/8 inches are not relevant for this problem). Therefore, as the crank is turned 10 revolutions clockwise, the entire screw will advance (move away from the crank) \( \frac{10}{16} \) inch, while the carriage will retreat (move towards the crank) \( \frac{10}{18} \) inch. The overall carriage motion is then:

\[
\delta x = \frac{10}{16} - \frac{10}{18} = \frac{90 - 80}{144} = \frac{5}{72} = 0.0694 \text{ in}
\]  

(2)

BTW—although not asked for—it is interesting to compute the effective “pitch” of the differential screw. Since 10 revolutions result in \( \frac{10}{144} \) in displacement, one revolution would cause \( \frac{1}{144} \) in displacement. The effective pitch would therefore be 144, equivalent to a single screw with 144 threads/in.

This is a pretty “fine” pitch, and would be difficult to obtain with a single thread.