

Chapter 6 HW Solution

Problem 6.2: The reciprocating flat-face follower motion is a rise of 2 in with SHM in 180° of cam rotation, followed by a return with SHM in the remaining 180° . The prime (base) circle radius $R_o = 2$ in, and there is *NO OFFSET*.

(a) Find the displacement functions $y(\theta)$ for the full motion and plot the displacement diagram using MATLAB. Use units of DEG for the plot.

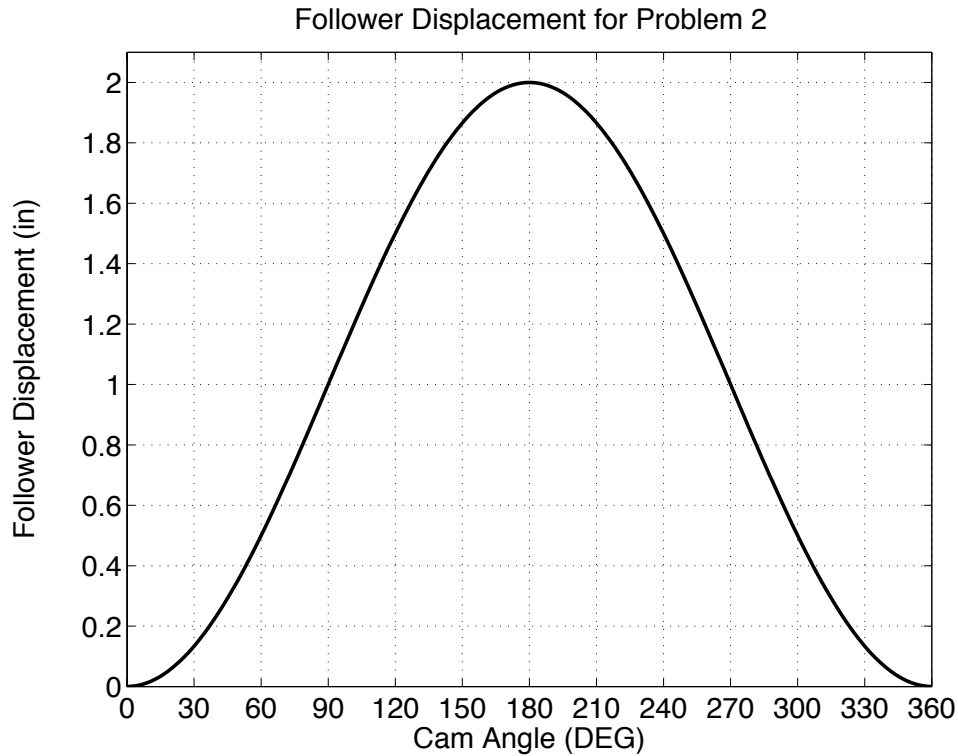
For the rise (segment 1), from text Figure 6.14 and equation (6.12a), where $L_1 = 2$ and $\beta_1 = \pi$, we have

$$y_1 = \frac{L_1}{2} \left(1 - \cos \frac{\pi\theta}{\beta_1} \right) = (1 - \cos \theta), \quad 0 \leq \theta \leq \pi \quad (1)$$

For the return (segment 2), from text Figure 6.17 and equation (6.15a), where $L_2 = 2$ and $\beta_2 = \pi$, we have

$$y_2 = \frac{L_2}{2} \left(1 + \cos \frac{\pi\theta}{\beta_2} \right) = (1 + \cos \theta), \quad 0 \leq \theta \leq \pi \quad (2)$$

To get the complete displacement function one simply evaluates (1) and (2) for $0 \leq \theta \leq \pi$, then concatenates both segments together (removing the common point at the segment boundary). The plot is shown below:



(b) Assuming the follower has a circular cross-section, how large must this follower radius be to accommodate the contact point?

The distance s from the follower axis to the contact point is simply equal to y' . Therefore all we need to is to find $\max(|y'|)$. Since both rise and return are “equal,” we can simply examine the rise. From text equation (6.12b), the follower “speed” y' is given by

$$y' = \frac{\pi L}{2\beta} \sin \frac{\pi\theta}{\beta} \implies \max|y'| = \frac{\pi L}{2\beta} = \frac{\pi(2)}{2(\pi)} = 1 \quad (3)$$

Therefore the follower radius must be $r = 1$ in (the corresponding diameter is 2 in)

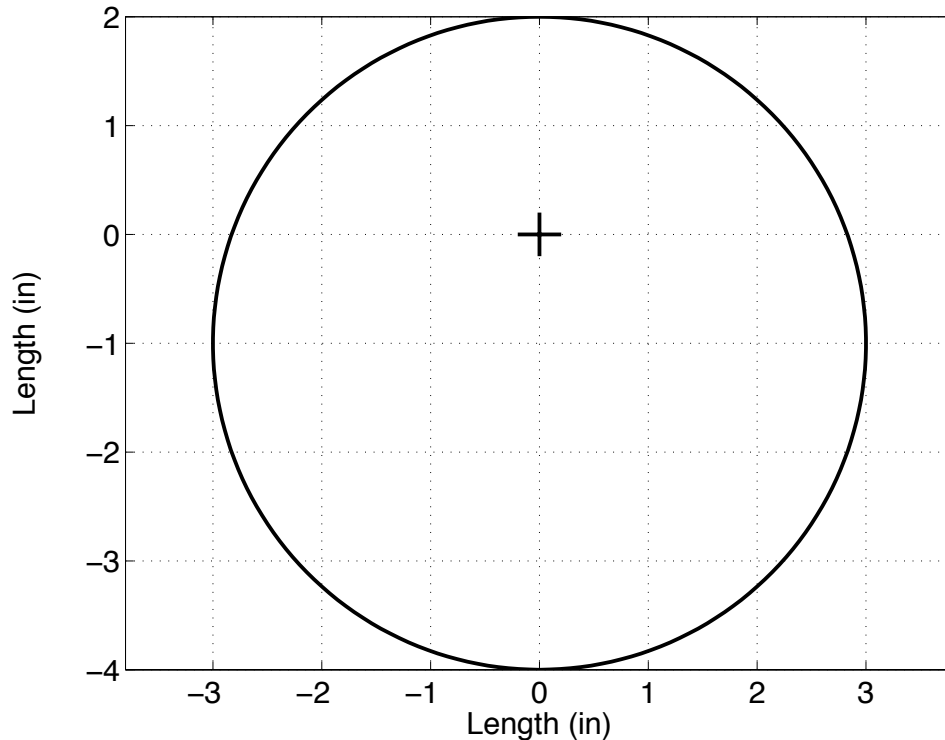
(c) Find and plot the profile that will accomplish this motion. Use 1° steps for cam angle θ .

I used the `camprofile.m` functions I wrote for this purpose. The format for this functions is:

```
FUNCTION [xc,yc] = camprofile(Ro,y,yp,theta,dir_flag)
```

where R_o is base circle radius, y and y_p are displacement y and y prime, θ is cam angle (RAD), and `dir_flag` is 'CCW' for this cam (the only change from CW to CCW is that the x component of cam profile coordinates must be negated—this produces a “mirror image” cam). Ironically, for the circular cam here it doesn't matter!

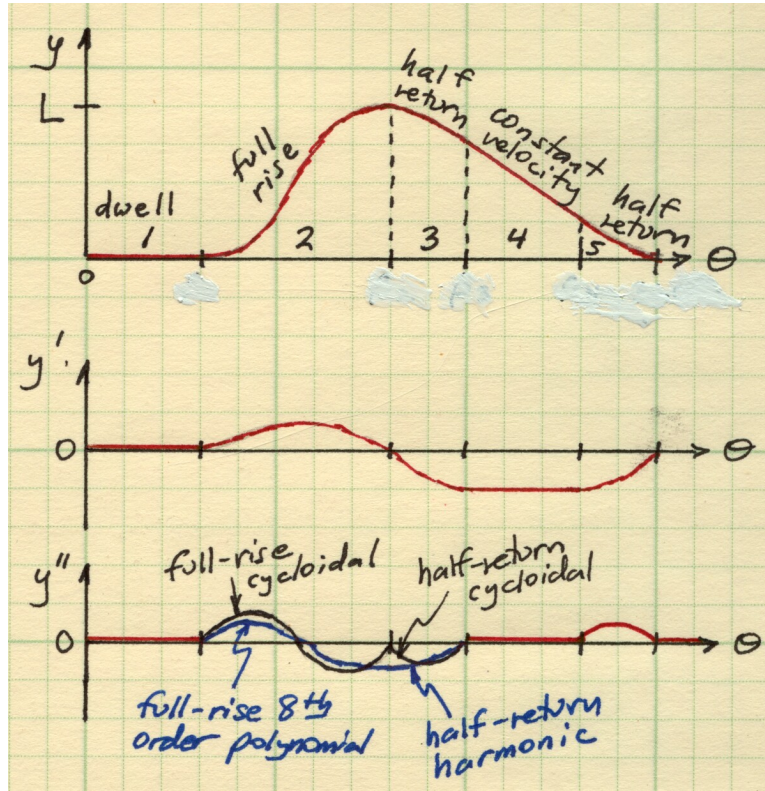
The displacement is shown below: it is a circle. That is because of the Simple Harmonic Motion. The “cross” indicates the axis of rotation. The vertical follower axis is at the top of the cam.



Problem 6.7: This problem is similar to text Example 6.2. The motion is composed of:

1. an initial dwell
2. a full-rise motion
3. a half-return to a uniform velocity
4. a uniform velocity segment
5. a half-return back to rest

(a) Plot the position y (in), velocity $\dot{y}$ (in/s), and acceleration $\ddot{y}$ (ft/s²) vs cam angle θ (DEG) for the complete motion. Since this thing is said to be a “high-speed” cam we need to keep the 2nd kinematic coefficient y'' continuous between motion segments. The sketch on the next page shows two possibilities for segments 2 + 3 for the overall motion. I’ve drawn in what we know: static initial dwell segment 1, and constant-velocity segment 4.



We need to have zero acceleration and velocity at the start of segment 2, zero velocity at the end of segment 2, and zero acceleration (and a velocity match) at the end of segment 3. Note that acceleration y'' need NOT be zero at the 2/3 boundary). This can be achieved in two ways:

- (i) Cycloidal full rise, followed by cycloidal half-return ($y'' = 0$ at segment 2/3 boundary)
- (ii) 8th-order full rise, followed by half-harmonic return ($y'' \neq 0$ at segment 2/3 boundary)

Comparison of text Figures 6.15 and 6.16 shows that the 8th-order full rise has lower peak acceleration (or y'') than the cycloidal. So I'll use the 8th-order polynomial for the full rise of segment 2, and a half-harmonic return for segment 3. Segment 5 must be a cycloidal half-return (zero acceleration at each end).

Constraints. The various motion constraints are given below. We need both the rise L and the duration β for each motion segment.

Segment 1: We know that the initial dwell is of duration 60° , so for segment 1 we have

$$\boxed{\beta_1 = 60^\circ = 1.0472 \text{ rad}} \quad (4)$$

and of course

$$y_1 = 0 \quad (5)$$

$$y_1' = 0 \quad (6)$$

$$y_1'' = 0 \quad (7)$$

Segments 2–3: This is an 8th-order polynomial full-rise (text Figure 6.16 and equations 6.14), which will “automatically” have zero initial velocity and acceleration, and terminate with zero velocity and negative acceleration, followed by a half-harmonic return (text Figure 6.21(a) and equations 6.20), which starts with zero velocity and negative acceleration and terminates with zero acceleration and negative velocity.

(a) We need to match acceleration at the junction between the two. The acceleration at the end of the 8th-order (segment 2) is

$$y_2''(1) = \frac{L_2}{\beta_2^2}(-5.2683) = -\frac{13.1708}{\beta_2^2} \quad (8)$$

and the acceleration at the beginning of half-harmonic (segment 3) is

$$y_3''(0) = -\frac{\pi^2 L_3}{4\beta_3^2} = -\frac{2.4674L_3}{\beta_3^2} \quad (9)$$

Equating these two expressions, we get

$$\boxed{\frac{L_3\beta_2^2}{\beta_3^2} = 5.3379} \quad (10)$$

Segments 3–4: The velocity at the end of segment 3 and the beginning of segment 4 must agree. Segment 4 is “constant-velocity” (CV) at 40 in/s for 1 in. This requires

$$t_4 = \frac{1 \text{ in}}{1} \frac{\text{s}}{40 \text{ in}} = 0.025 \text{ s} \quad (11)$$

At 400 rpm, we have

$$\beta_4 = \omega t_4 = \frac{400 \text{ rev}}{\text{min}} \frac{\text{min}}{60 \text{ s}} \frac{360 \text{ deg}}{\text{rev}} * 0.025 \text{ s} = 60^\circ = 1.0472 \text{ rad} \quad (12)$$

so

$$\boxed{\beta_4 = 1.0472 \text{ (} 60^\circ \text{)}} \quad (13)$$

Also, from the given constant velocity of -40 in/s we have

$$y_4' = \frac{\dot{y}_4}{\omega} = -0.9549 \text{ in/rad} \quad (14)$$

The velocity at the end of half-harmonic segment 3 is

$$y_3'(1) = -\frac{\pi L_3}{2\beta_3} \text{ which must equal } -0.9549 \quad (15)$$

so by equating these we end up with

$$\boxed{L_3 = 0.6079\beta_3} \quad (16)$$

Segments 4–5: The cycloidal half-return for segment 5 (text Figure 6.23(b) and equations 6.25) will inherently have zero final position, velocity, and acceleration. However, we must match the initial negative velocity of segment 5 to the constant negative (-0.9549) velocity of segment 4. Skipping some of the details, this yields

$$\boxed{L_5 = 0.4775\beta_5} \quad (17)$$

Other Relationships: Some of the other constraints are the given displacement of the initial full-rise:

$$\boxed{L_2 = 2.5 \text{ in}} \quad (18)$$

Also, since the CV return segment has a length of 1 in, we have

$$\boxed{L_3 + L_5 = 2.5 - 1 = 1.5 \text{ in}} \quad (19)$$

Finally, all the cam durations must sum to 2π radians, so we have

$$\boxed{\beta_2 + \beta_3 + \beta_5 = 2\pi - \beta_1 - \beta_4 = 4.1888 \text{ rad (} 240^\circ \text{)}} \quad (20)$$

Equation Summary. Taking all of the “boxed” equations into account, I ended up with five equations in five unknowns ($L_3, L_5, \beta_2, \beta_3, \beta_5$). Four of the equations are linear; the fifth is nonlinear. They are not *too* hard to solve manually (or maybe I’m just getting too old); the results I got are:

$$L_3 = 0.0814 \text{ in} \quad (21)$$

$$L_5 = 1.4186 \text{ in} \quad (22)$$

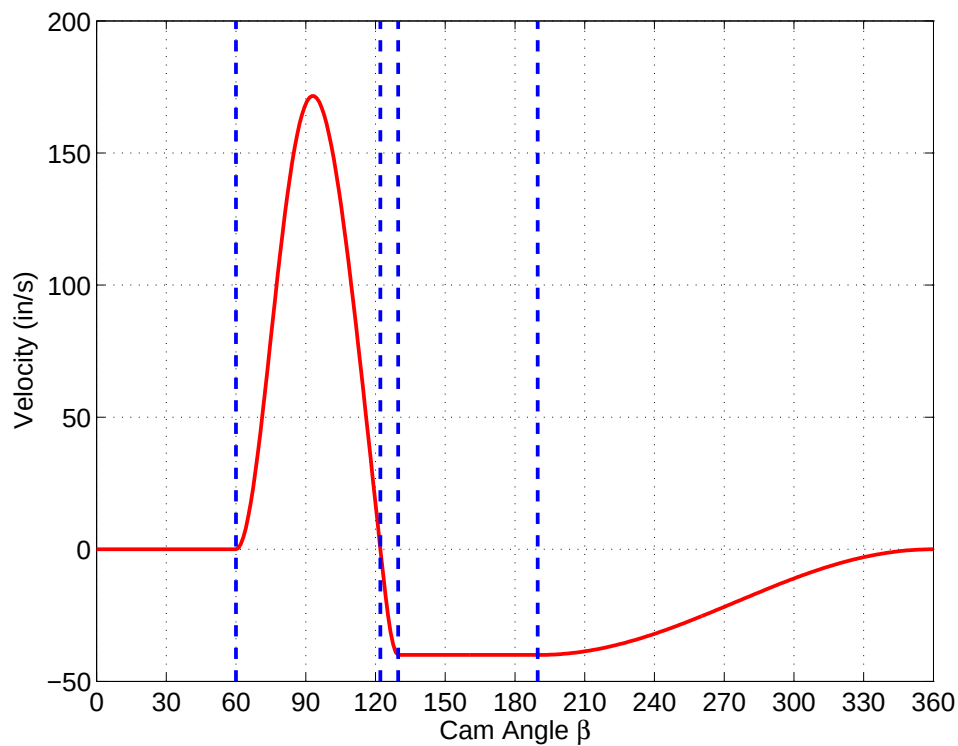
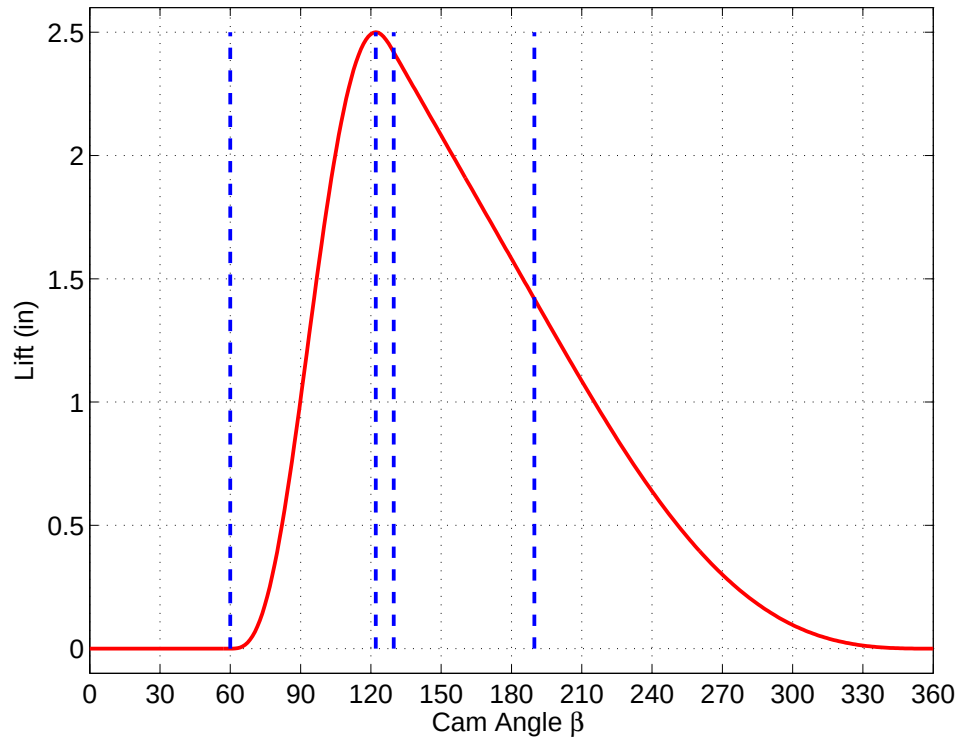
$$\beta_2 = 1.0850 \text{ rad (} 62.1084^\circ \text{)} \quad (23)$$

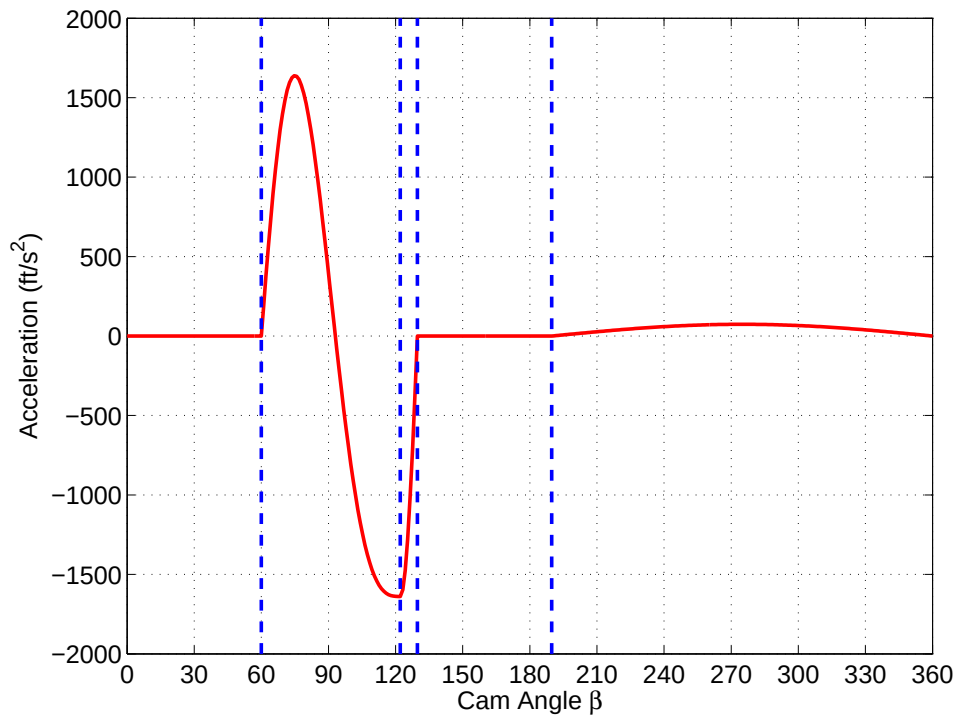
$$\beta_3 = 0.1339 \text{ rad (} 7.6721^\circ \text{)} \quad (24)$$

$$\beta_5 = 2.9709 \text{ rad (} 170.2195^\circ \text{)} \quad (25)$$

The final motion segment is *MUCH* longer in duration than the others—almost half the rotation angle—pretty surprising. Of course I didn't create this problem...

Displacement Diagrams. Preparing all the motions $(y, \dot{y}, \ddot{y})$ in MATLAB (remember to multiply y' and y'' by ω and ω^2) is pretty tedious, but I did it, and you should have, too. The plots are shown below, with dashed lines marking the segment boundaries. Again, the “shape” of the resulting motion is surprising—the 3rd segment is very short, and the 5th is quite long.





Note that the units of acceleration are ft/s^2 , with the peak value being 1638 ft/s^2 , or about 50 g's.

(b) Assuming the follower has a circular cross-section, how large must the follower radius be to accommodate the contact point?

The distance from the follower axis to the contact point is simply y' , so the follower radius must be equal to $\max(|y'|)$. From the velocity plot on the previous page, the maximum velocity $|y'|$ occurs during segment 2. The “velocity” function for segment 2 is given in text equation (6.14b). We must find the maximum of this equation, so differentiate it with respect to θ and set equal to zero. However, this derivative is simply y'' , which is given in text equation (6.14c). The roots of (6.14c) can be found using the MATLAB function `roots`. This yields (note that the 0^{th} and 2^{nd} order coefficients are zero):

```
>> roots([143.4132 -571.6053 801.9465 -415.608 0 36.5853 0])
```

```
ans = 0
      1.4489
      1.1215 + 0.3558i
      1.1215 - 0.3558i
      0.5326
     -0.2388
```

These are values of θ/β , and since $0 \leq \theta/\beta \leq 1$ the only admissible result is

$$\frac{\theta}{\beta} = 0.5326 \implies \theta = 0.5326\beta_2 = 0.5774 = 33.0816^\circ \quad (26)$$

Adding the angle of (26) to β_1 yields an angle of 93.0816° where the segment 2 acceleration is zero, which seems to agree with the plot above. Now evaluate y' using the normalized angle $\theta/\beta = 0.5326$ to find the maximum y' , which is (one can use MATLAB function `polyval` to evaluate this polynomial)

$$(y')_{\max} = \frac{L_2}{\beta_2} [18.29265(0.2417)^2 - 103.902(0.2417)^4 + \dots] = 4.0975 \text{ in} \quad (27)$$

So the follower radius must be 4.0975 inches.

By the way, the “easy” way to do this part is to realize that from the previous figure we have (from MATLAB) the series of samples for y'_2 , and can simply find the maximum value from MATLAB, like this:

In my analysis (stepsize of 1°), the vector for y'_2 is MATLAB variable `yp2`, and the maximum value is:

```
>> max(yp2)
```

```
ans = 4.0959
```

Due to the finite stepsize, this “approximate” value of $y' = 4.0959$ is not *quite* as accurate as the “analytical” value of 4.0975, but it is pretty darned close (and *much* easier).

(c) Find and plot the cam profile. To avoid any cusps on the cam profile, the radius of curvature $\rho > 0$. Apply text equation (6.27) and find the minimum base circle $(R_o)_{min}$. Round $(R_o)_{min}$ up to the nearest inch, and use that value for R_o .

From text (6.27), the radius of curvature of the cam profile can be written as

$$\rho = R_o + y + y'', \text{ hence at } \rho = 0 \text{ we have} \quad (28)$$

$$R_o = -y - y'' \quad (29)$$

The minimum value of base circle radius $R_o = \max(-y - y'')$. Got that? Taking the easy way out—since I have the MATLAB functions computed for all segments, just use those to find the maximum:

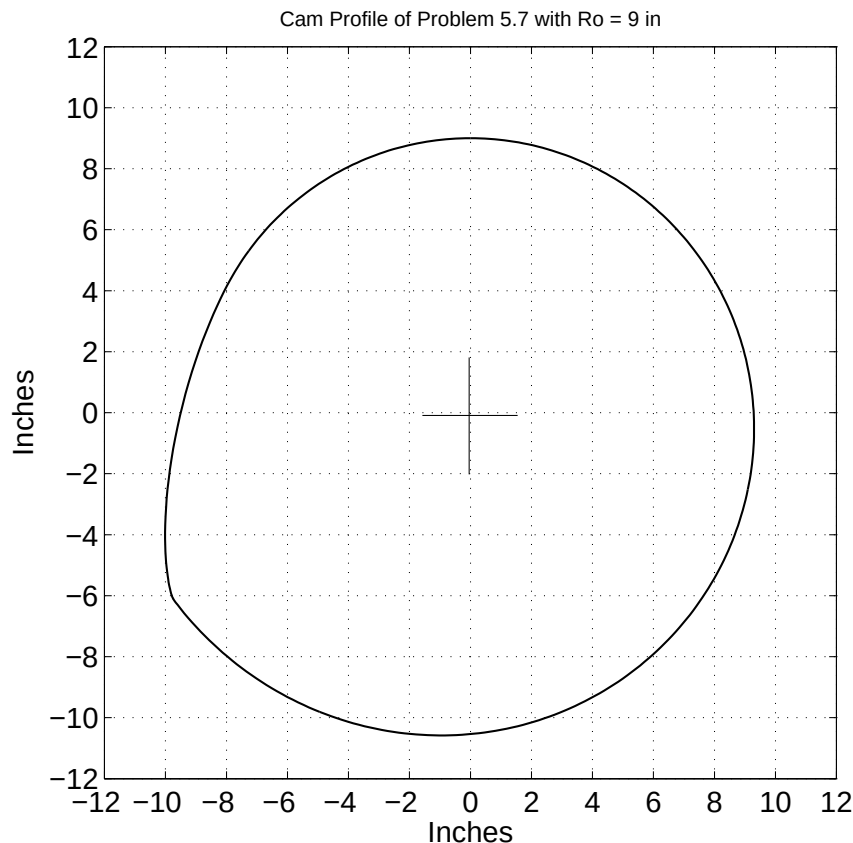
```
>> max(-y-ypp)
```

```
ans = 8.7113
```

Rounding up to the nearest inch, this yields a base circle radius of

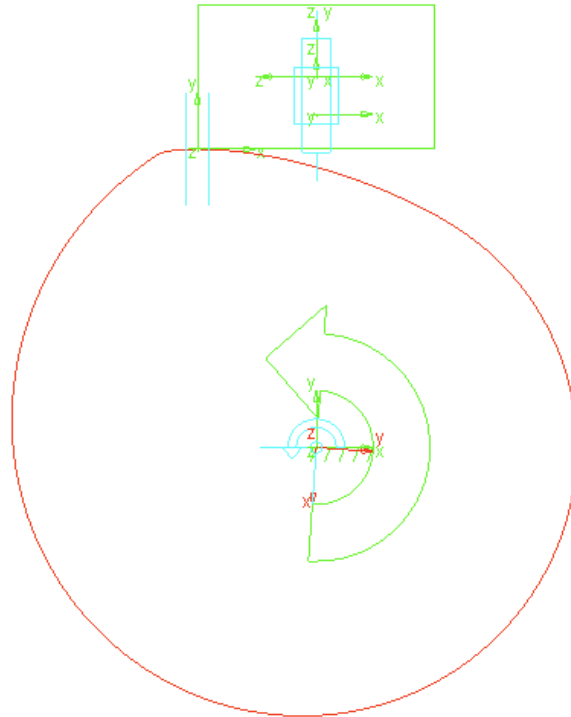
$$\boxed{R_o = 9 \text{ in}} \quad (30)$$

The cam profile with $R_o = 9$ in is shown below. The minimum ρ occurs on the SW corner.

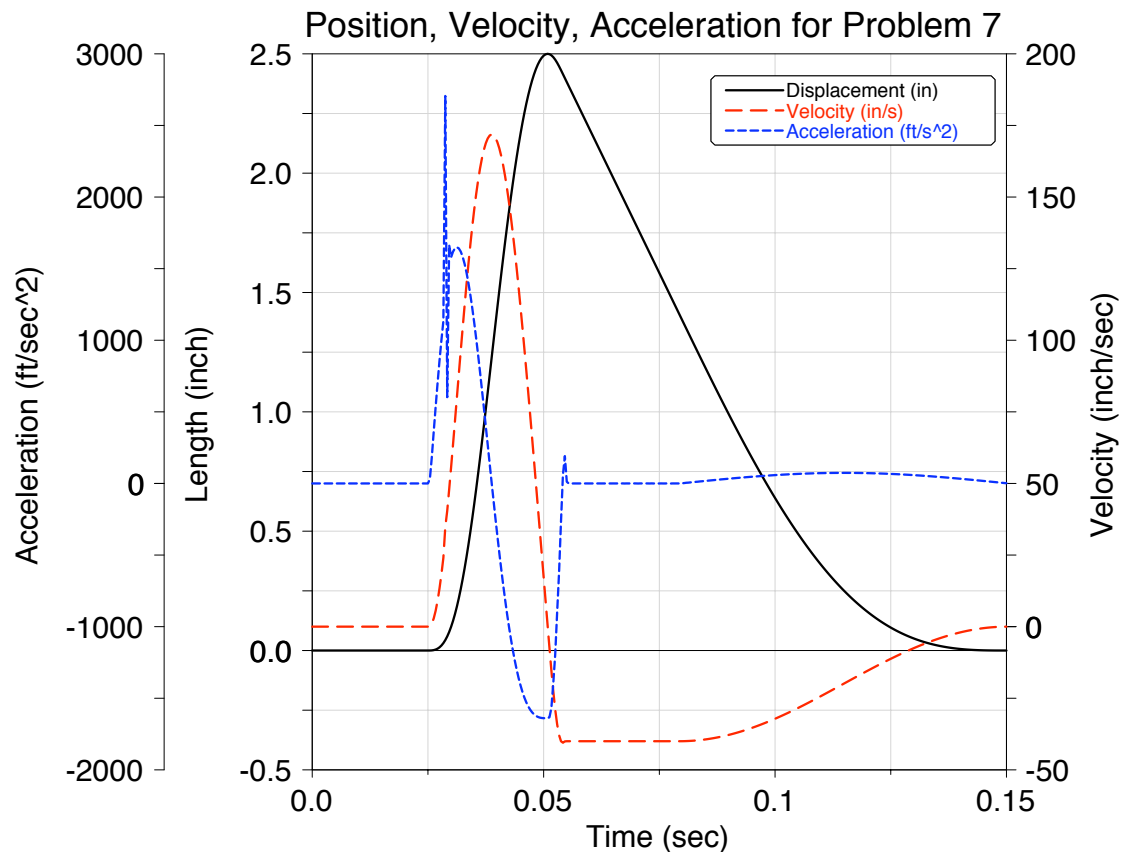


(d) Using the numerical data from parts (b) and (c), construct an ADAMS model of the cam and follower. Verify that the follower displacement, velocity, and acceleration agree with the desired behavior.

I followed the procedure I showed you in class: here is a screenshot of my cam at the point where the contact point is right at the edge of the follower (I changed the ADAMS screen background to white):



An ADAMS plot of follower position, velocity, and acceleration is shown below. Not sure about that acceleration jitter...



Problem 6.9: If the cam of Problem 6.7 is driven at constant speed, determine the time (duration) of the dwell and the maximum and minimum velocity and acceleration of the follower for the cam cycle.

(a) The dwell has a angular duration of 60° ; this is the same angular duration as the constant-velocity segment. In Problem 6.7 we found the corresponding CV segment duration; it's the same for the dwell, *i.e.*

$$t_{dwell} = 0.025 \text{ sec} \quad (31)$$

(b) To draw the plots, I had to compute MATLAB vectors for y , $\dot{y}$, and $\ddot{y}$; finding the max and min of these is easy (MATLAB “max” and “min” functions); their values are:

$$\dot{y}_{max} = 171.5698 \text{ in/s} \quad (32)$$

$$\ddot{y}_{max} = 1.9660 \times 10^4 \text{ in/s}^2 \text{ or } 1638.3 \text{ ft/s}^2 \quad (33)$$

Note that these are “approximate” values in that my stepsize for computing was about 1 degree. The analytical max/min values may be slightly different.

However, we’ve already computed the exact maximum in part (b) while finding the required follower radius. Recall that value was

$$y'_{max} = 4.0975 \text{ in} \quad (34)$$

So a more accurate value for maximum velocity is

$$\dot{y}_{max} = y'_{max}\omega = 171.6342 \text{ in/s} \quad (35)$$

which is slightly larger.

Doing the same thing for acceleration yields:

```
>> poly8jerk = [860.4792 -2858.0265 3207.786 -1246.824 0 36.5853];
>> roots(poly8jerk)
```

```
ans = 1.2235
      1.0000
      1.0000
      0.2417
     -0.1438
```

Again, the only admissible value is 0.2417, so

```
>> tjmax = ans(4);
>> yppmax = polyval(poly8acc,tjmax);
```

```
yppmax = 11.2086
```

```
>> yddmax = yppmax*w^2
```

```
yddmax = 1.9667e+04
```

```
>> yddmax/12
```

```
ans = 1.6389e+03
```

So the revised maximum acceleration value is

$$\ddot{y}_{max} = 1638.9 \text{ ft/s}^2 \quad (36)$$

which is slightly larger than the previous value.