

Cam Design Example

1 Problem Statement

Design a disk cam and reciprocating flat-face follower such that the follower rises 1 inch during the first 120° of cam motion, then returns during the next 240° of cam motion. Use Simple Harmonic Motion for both rise and return. Also find the minimum radius of the cylindrical follower.

2 Displacement Functions

The motion consists of two segments: (1) a full-rise, and (2) a full-return. Since we using SHM, text Figures 5.14 and 5.17 (and their associated equations) are relevant. Both y and y' are used in finding the cam profile; y' is used in finding the follower face radius, and y and y'' are used in finding the minimum base circle radius.

2.1 Full-Rise Motion

The displacement function $y(\theta)$ for the full-rise SHM motion is

$$y = \frac{L}{2} \left(1 - \cos \frac{\pi\theta}{\beta} \right), \quad (1)$$

and the first and second kinematic coefficients $y'(\theta)$ and $y''(\theta)$ are

$$y' = \frac{\pi L}{2\beta} \sin \frac{\pi\theta}{\beta} \quad y'' = \frac{\pi^2 L}{2\beta^2} \cos \frac{\pi\theta}{\beta} \quad (2)$$

where

$$\begin{aligned} L &= 1 \text{ inch} \\ \beta &= 120^\circ = \frac{120\pi}{180} = \frac{2\pi}{3} \\ \theta &= 0^\circ \dots 120^\circ \text{ (will convert to RAD during evaluation)} \end{aligned}$$

2.2 Full-Return Motion

The displacement function for the full-return SHM motion is

$$y = \frac{L}{2} \left(1 + \cos \frac{\pi\theta}{\beta} \right), \quad (3)$$

and the first and second kinematic coefficients $y'(\theta)$ and $y''(\theta)$ are

$$y' = -\frac{\pi L}{2\beta} \sin \frac{\pi\theta}{\beta} \quad y'' = -\frac{\pi^2 L}{2\beta^2} \cos \frac{\pi\theta}{\beta} \quad (4)$$

where

$$\begin{aligned} L &= 1 \text{ inch} \\ \beta &= 240^\circ = \frac{240\pi}{180} = \frac{4\pi}{3} \\ \theta &= 0^\circ \dots 240^\circ \text{ (will convert to RAD during evaluation)} \end{aligned}$$

These motions will be evaluated using MATLAB and used in various sections to come. We will also need the second kinematic coefficient y'' later for calculating the minimum base circle radius $(R_o)_{\min}$.

3 Follower Radius

The contact point is a distance s away from the follower axis. Assuming a cylindrical follower with circular cross section, the radius of the follower must be equal to s_{\max} .

Since from the cam geometric analysis we know that

$$s = y', \quad (5)$$

the minimum allowable cam radius is simply equal to the maximum (or minimum) value of y' , hence

$$r_{\text{follower}} = \max(|y'|) \quad (6)$$

The largest contact point offset will occur at the **peak follower velocity**. Since the rise occurs in less cam angle (and hence less time) than the return, the extremum of y' will occur during the rise.

Examination of the plot of y' in text Figure 5.14 shows that the maximum value occurs at the midpoint, when $\theta/\beta = 0.5$. From equation (2a) we have

$$y' = \frac{\pi L}{2\beta} \sin \frac{\pi\theta}{\beta}, \quad (7)$$

for which the maximum value is

$$(y')_{\max} = \frac{\pi L}{2\beta} = \frac{(\pi)(1)}{(2)(2\pi/3)} = 0.75 \text{ inch} \quad (8)$$

So the follower must have radius of 0.75 inch, or a diameter of 1.5 inches.

4 Base Circle Radius

The *base circle* of the cam must be large enough so the cam profile has no cusps, that is, the radius of curvature of the cam profile $\rho > 0$.

4.1 Analytical Radius of Curvature

Although I have not derived it in class, text equation (5.33) relates the radius of curvature ρ , base circle radius R_o , follower displacement y , and follower second kinematic coefficient y'' :

$$\rho = R_o + y + y''. \quad (9)$$

This relationship allows you to calculate the minimum base circle without determining the cam profile, which is what you need. Otherwise you would need to select a base circle, find the cam profile, evaluate it for curvature (*i.e.* cusps), select another base circle, and try again.

A cusp occurs when $\rho = 0$, hence the minimum base circle radius to avoid cusps is

$$R_o = (-y - y'')_{\max} \quad (10)$$

If you wish to maintain cam radius of curvature equal to a specified ρ_{\min} , then the base circle is given by

$$R_o = \rho_{\min} + (-y - y'')_{\max} \quad (11)$$

I will use a MATLAB script to design the cam, and one portion of the script will compute equation (10) to find the base circle radius.

5 MATLAB Script for Cam Design

I will create a MATLAB script for design of this cam. It will compute the displacement functions and the necessary kinematic coefficients. It will also determine the minimum follower radius and the minimum base circle radius, then allow user input to override those values.

I will put this script online to serve as an aid for you in the homework problems.