Automotive Cam Problem
(worth another 10 points)

1 Introduction

After the text cam problems have given us these big, weirdly-shaped cams, I thought it would be interesting to design a cam with similar specifications to those used in typical automotive internal-combustion engines. For those of you who don’t know: the purpose of a cam (or camshaft) in an IC engine is to operate (open and close) the intake and exhaust valves.

1.1 Cam Specifications

Automotive cams are typically specified using two quantities

- **Lift** — the total amount of follower displacement \( L \)
- **Duration** — the amount of camshaft rotation during which the valve is open

1.1.1 Definition of Duration

The specification of lift is clear, but duration is not. When is the valve defined as being open (or closed)? One convention that is sometimes used is the 0.050 inch point, *i.e.*

\[
\begin{align*}
\text{The open and closed points are where the follower displacement is 0.050 inch, hence} \\
\text{the given duration is the cam angle between those points}
\end{align*}
\]

This duration will be slightly less than the total cam angle \( \beta \); this will be discussed in more detail in Section 3.

2 Problem Statement

You are to design three cams, all with the same lift and duration.

2.1 Cam Specifications

- Lift \( L = 0.400 \) inch
- Duration 200° (between the 0.050-inch open & closed points)

These are values typical of an automotive cam.

2.2 Displacement Functions

The three cams are the following

1. Harmonic rise and return (text equations 5.18 and 5.21)
2. Cycloidal rise and return (text equations 5.19 and 5.22)
3. 8\(^{th}\) order polynomial rise and return (text equations 5.20 and 5.23)

The rise and return for each cam should have the same \( \beta \); they will be symmetric about the maximum rise point.
2.3 Required Work

For each cam design do the following:

1. Find the $\beta$ for the rise and return (will be equal)

2. Write a MATLAB script that does the following:
   - Finds $y, y', y''$ vs $\theta$ for the complete motion
   - Finds the minimum follower radius $r_{\text{min}} = (y')_{\text{max}}$
   - Finds the minimum base circle radius $R_0 = (-y - y'')_{\text{max}}$

3. Plot the displacement, 1st and 2nd kinematic coefficient ($y, y', y''$) vs cam angle $\theta$

4. Select $R_0 = 0.5$ inch and plot the cam profile.

Which cam would you select for high-speed operation? Why?

2.4 Extra Credit (another 5 points)

Build an ADAMS model of the cam you would select for high-speed operation.

3 Finding the Motion Duration $\beta$

The total angle duration $\beta$ for each rise and return will be greater than one-half of the specified 200° cam duration. This is due to the 0.050 inch open/closed points specified for this cam. The situation is shown graphically in Figure 1 below:

![Figure 1: The difference between 100° duration and the actual $\beta$.](image)

The horizontal dashed line is drawn at $0.125L = 0.050$ inch. To find $\beta$ you must find the value of $\theta$ where $y = 0.125L$. This will be different for each cam, and is found by setting the displacement equation equal to $0.125L$ and solving for $\theta$. Actually, it will be easier to solve for the “dimensionless” value of $\theta/\beta$. 
As an example, for the \textbf{cycloidal} motion function we have (text equation (5.19a)),

\begin{equation}
    y = L \left( \frac{\theta}{\beta} - \frac{1}{2\pi} \sin \frac{2\pi\theta}{\beta} \right) \text{ inch}
\end{equation}  \tag{1}

Setting \( y = 0.050 = 0.125L = L/8 \) in (1) yields

\begin{equation}
    \frac{L}{8} = L \left( \frac{\theta}{\beta} - \frac{1}{2\pi} \sin \frac{2\pi\theta}{\beta} \right)
\end{equation}  \tag{2}

If we define \( x = \frac{\theta}{\beta} \), then (2) can be expressed as:

\begin{equation}
    f(x) = x - \frac{1}{2\pi} \sin(2\pi x) - \frac{1}{8} = 0
\end{equation}  \tag{3}

Since (3) is nonlinear, it is not easy to solve. However, there is a \textsc{Matlab} function perfect for this: \texttt{x=fzero(fn,x0)}. Parameter \texttt{fn} is a \textbf{function handle} (remember those?) to the function of (3), and \texttt{x0} is your initial “guess.” You can make a guess of \texttt{x0 = 0.1} and be plenty close (\( \theta/\beta \) goes from 0 \( \rightarrow \) 1).

Actually the \textbf{harmonic} function can be solved manually, but it might be a good test of the \texttt{fzero} function to try that one also. The 8\textsuperscript{th} order polynomial would be a challenge without \texttt{fzero}.  

\[ \text{ } \]