LEGO Motor Calibration

1 Rationale

To develop a closed-loop control system, we need information on the behavior of the components which comprise that system. For your LEGO robots, there are at least two critical components:

- Motors
- Sensors

This assignment deals with the beginnings of motor calibration.

2 Motor Modeling

2.1 Inputs

2.1.1 Armature Voltage

One input is **armature voltage** v_a, or in our case, **PWM drive intensity** (the "duty cycle" in the Interactive-C `motor()` function.

2.1.2 Disturbance Torque

Another "input" which you may not have considered is **disturbance torque** T_d. This is really the output torque of the motor when it is driving something (as opposed to freely running with no load.

2.2 Motor Equations

You should have been exposed to analysis of an armature-controlled DC motor in your Control Systems class. Following is a summary.

2.2.1 Armature Voltage Equation

The two terminals on the motor are connected to the motor armature; an electrical equation for this circuit is:

$$v_a - L_a \frac{di_a}{dt} - i_a R_a - K_b \omega_m = 0, \tag{1}$$

in which

$$\begin{aligned}
v_a &= \text{applied armature voltage in V} \\
L_a &= \text{armature inductance in H (usually neglected)} \\
i_a &= \text{armature current in A} \\
R_a &= \text{armature resistance in } \Omega \\
K_b &= \text{back EMF constant in V-s/rad} \\
\omega_m &= \text{motor speed in rad/s}
\end{aligned}$$

2.2.2 Torque Generated

The equation for the torque generated (this type of relationship is true for *any* type of electromechanical actuator) is:

$$T = K_t i_a, \tag{2}$$

in which

$$\begin{aligned} T &= \text{torque generated in N-m} \\ K_t &= \text{torque constant in N-m/A.} \end{aligned}$$

2.2.3 Motor Torque Equation

Applying Newton's 2^{nd} Law for the armature yields

$$T = J_m \dot{\omega}_m + B_m \omega_m, \tag{3}$$

in which

$$\begin{aligned} J_m &= \text{armature inertia in kg-m}^2 \\ B_m &= \text{motor viscous friction coefficient in N-m-s} \end{aligned}$$

2.3 Motor Behavior

Now, there are a *bunch* of coefficients in (1)–(3) which we have **NO HOPE** of finding. Nevertheless, in steady-state one can show that

$$\omega_m = \left(\frac{K_t}{B_m R_a + K_b} \right) v_a, \tag{4}$$

that is, as you increase motor voltage the motor speeds up; this should be a **LINEAR** relationship.

Now, as I showed in class, for the LEGO gearmotors and the "pseudo-PWM" drive of the `motor()` function, this relationship is **NOT** linear. But we need to quantify it.

3 Your Assignment for Monday, February 18

Each team should do the following, and submit all relevant documentation in class on Monday.

3.1 Data Collection and Plotting

3.1.1 Data Collection

Obtain motor speed data for a range of duty cycles. You may wish to count revolutions of a gear-reduction manually, or try to automate this process using an IR reflectance sensor. Try several motors to see how they vary.

3.1.2 Plotting

Plot motor speed (rad/s) *vs* duty cycle (integer %) for each motor you test. Also plot the inverse: duty cycle *vs* motor speed.

3.2 Curve Fitting

If you wish, make an attempt at fitting a curve to both of these plots. Note that in Interactive-C integer operations are much faster than floating point, and mathematical functions are quite slow.