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Inertial Data for the WAM arm.

This document presents the inertial data for the WAM robotic arm (both the 4- and 7-dof version) in support of computed-torque control techniques.

Relevance and Approximation of "Reflected Inertia."

Except for (generally impractical) direct-drive robotic arms, all multi-serial-link robotic arms have drive inertias whose main effect may be approximated as "reflected" inertias. Reflected inertias are only approximations because of the assumption that there is no cross-product interaction between the spinning drive components and the (generally rotating) frames in which they are embedded. Reflected inertias are amplified by the square of the drive-reduction-ratios which generally range from 30 – 1,000 for practical robotic arms, so the reflected inertias generally range from 1,000 – 1,000,000, easily contributing most of the inertia of a robot. It is interesting to note that most text books on robotics deemphasize or dismiss the contributions of reflected inertias even though they generally overwhelm the inertias of the moving links. The WAM is unique is that its ratios were selected to balance the two inertia sources for optimal backdrivability and so both inertia sources are important.

Relevance and Treatment of "Differential" Mechanisms.

Also, many, if not most, robotic arms minimize moving inertia through the use of drive mechanisms called "differentials" which allow a pair of motors to be placed on one side of two perpendicular, intersecting axes, such as at shoulders and wrists, thereby minimizing the moving inertia and of those motors. However, differentials also require special treatment when applying the approximation of "reflected" inertias because of the 2x2 motor-joint couplings, especially in the case of a high-backdrivability robot like the WAM. The couplings are given by the middle 2x2 transformations of Equation 1 for the shoulder, where $n_3 = 1.6800$, and Equation 2 for the wrist, where $n_6 = 1.0000$. For any pair of computed Joint torques the kinematically equivalent Motor torques are easily calculated as the sum of 2 polynomials with constant coefficients.

$$\begin{pmatrix} M\tau_1 \\ M\tau_2 \\ M\tau_3 \\ M\tau_4 \end{pmatrix} = \begin{pmatrix} \frac{-1}{N_1} & 0 & 0 & 0 \\ 0 & \frac{1}{2N_2} & \frac{-n_3}{2N_2} & 0 \\ 0 & \frac{-1}{2N_2} & \frac{-n_3}{2N_2} & 0 \\ 0 & 0 & 0 & \frac{1}{N_4} \end{pmatrix} \begin{pmatrix} J\tau_1 \\ J\tau_2 \\ J\tau_3 \\ J\tau_4 \end{pmatrix}$$

Equation 1: Arm Joint-to-Motor torque transformations

$$\begin{pmatrix} M\tau_5 \\ M\tau_6 \\ M\tau_7 \end{pmatrix} = \begin{pmatrix} \frac{1}{2N_5} & \frac{-n_6}{2N_5} & 0 \\ \frac{1}{2N_5} & \frac{n_6}{2N_5} & 0 \\ 0 & 0 & \frac{-1}{N_7} \end{pmatrix} \begin{pmatrix} J\tau_5 \\ J\tau_6 \\ J\tau_7 \end{pmatrix}$$

Equation 2 - Wrist Joint-to-Motor torque transformations

The inertial data are calculated and employed separately for Frames and Drives:

1. Link (or Frame) inertias that are associated by Coordinate Frame number (0-7).
2. Drive inertias that are associated by Motor number (1-7).

How to Use the Inertia Data

Generally, motor torques are calculated in the following sequence:

1. Use the Frame inertias in the generalized robot equation. Apply either LaGrange or numerical Newton-Euler methods to translate desired robot forces and torques described in world Cartesian coordinates into equivalent, computed, joint torques.
2. For axes driven by differentials at the shoulder and wrist, apply the 2x2 torque transformations to translate the computed joint torques into the kinematically equivalent ideal motor torques.
3. For each ideal motor torque, calculate an adjusted motor torque required to provide the appropriate additional motor-output torque that overcomes the drive inertias.

Frame Inertias (and Related Mass Data)

See the data in Table 2 through Table 11 that correspond to the model graphics in Figure 4 through Figure 13. Some of the data in the tables is not necessary for most computed-torque calculations. All units are in kilograms and meters unless otherwise indicated. Generally, 13 values are important:

- Mass.
- X, Y, Z location of the Center of Mass.
- Ixx through Izz (9 inertia-tensor values).

Drive Inertias

The motor-drive inertias are reported in Table 1. The data is derived from model geometries shown in Figure 14 through Figure 24. In these models, roller and ball bearings are generally separated into their stationary and moving components. The geometric information from these models is summarized in Table 23 along with the associated cable diameters and resulting intermediate and total drive ratios. The inertial data from these models, in which only one inertia tensor (bolded and underlined) value is relevant are given in Table 12 through Table 22. The additional inertia data can, for example, quantify corrections in the "reflected"-inertia assumptions though it is believed that these errors are well under 1%. Table 24 calculates the lumped pinion+cable inertia for each drive body and gives its equivalent inertia at the rotor end of the drive. Table 25 then combines the various drive-components for each motor-drive number.

Table 1 -- Drive Inertias for all WAM DOFs.

Motor Drive Number	Total Drive Inertia at Rotor (kg-m^2)	Drive Ratios	Total Reflected Inertia at Output (kg-m^2)
M1	0.00011631	42.00	0.205190
M2	0.00011831	28.25	0.094428
M3	0.00011831	28.25	0.094428
M4	0.00010686	18.00	0.034628
M5	0.00001685	9.70	0.001584
M6	0.00001745	9.70	0.001641
M7	0.00000142	14.93	0.000318

This draft document is a work in progress and not ready to be released for general use. Critical feedback is encouraged.

There are several one-page graphics that show an assembly (rigid body) with its associated coordinate frame and reports of mass parameters (mass, CG, and inertia tensor). Many of the components of the assembly are rendered as transparent to make clear what components are and are not included in the assembly. The numerical data is reported with the screen-shot graphic (to prevent any possibility of associating data to the wrong model). However the data is then repeated as text on the page following each graphic to allow easy copy-and-paste to a user's program.

Figure 1 shows the relationships between pulleys throughout the WAM. The present-day WAM has changed little since this artist's sketch was produced in 1987. In modern WAMs there are twin cables in each 2nd stage where the illustration shows only one cable, and the base motor has flipped orientation for better compactness. Figure 2 helps clarify the operation of the differential. It should be noted that the rotations of both differential-input pulleys are totally independent of the structure that supports them and (therefore) independent of the orientation of motor 4. One depends on the rotation of the M2 rotor and the other on the M3 rotor.

Figure 14 through Figure 16 are rigid-models required to calculate the "reflected" inertia of each of the 1st four DOFs of the WAM. In cases where there are sets of ball or roller bearings supporting a spinning body, we deleted ½ of the bearings as an approximation. For the elbow pulley-pinion, we can take an average of the case with all bearings installed (Figure 17) and the case of no bearings installed (**Error! Reference source not found.**).

Cable, pinion, and pulley diameter/radius geometric data given in Table 23 enable calculation of all transmission ratios including intermediate ratios required to associate the "grenades" of Figure 15 and elbow-pulley-pinion set of Figure 17 & **Error! Reference source not found.** with their associated joint inertias. The only ratio not given in this table (but given in this sentence) is the 1.68:1 ratio between the differential-input-pulley radii and the output pulley radius for calculating the J3 reflected inertias.

The (unreflected) motor rotor inertia (Izz only) of Figure 14 is identical for each motor of the 1st four DOFs of the WAM = 0.00010569 kg-m^2. Also for all 4 DOFs, this rotor drives a pair of identical Stage-1 cables (Sava Cable Part Number SN2047 in Table 23) that have a mass of 0.013 kg each, or 0.026 kg total. The radius to the centerline of these cables where they are wrapped onto the scalloped pinion of the rotor is 0.009 m (9 mm), and since the entire length of cable translates as the same velocity one can lump the both cable masses at the 9-mm radius. In this case the inertia of the cables is (0.026 kg)*(0.009 m)^2 = 0.00000178 kg-m^2, adding <2% to the rotor inertia. The combined inertia is then = 0.00010747 kg-m^2.

At least for the 1st three WAM motor drives, the 1st and 2nd stages are coupled through a pair of "grenades" (Figure 15) that have an 11-mm radius pinion at one end driven by a stage-1 cable and a 33-mm-radius pulley at the other that drives a pair of stage-2 cables.

Cautions:

While the robot inertias are given here, several modeling errors are known to exist:

1. Machining, plastics, and composite-layup tolerances, especially important with thin (WAM) structures.
2. Ceramic, anodize/Teflon-surface-coating densities not accounted for.
3. Density variances from (allowed) tolerances in alloy contents, plastics, ceramics, and composites.
4. Inability to model (especially stranded) wires and cables in any practical manner, especially their paths as they flex.
5. Lack of knowledge of the proprietary assembly of components, such as bearings, wires, electrical cable assemblies, connectors, etc.
6. Treating ball and roller bearings as if they had only an inner and an outer race, when, in fact the balls themselves belong to neither, have individual (unmodelled) spins, and their CGs travel neither with the inner race nor the outer race. Ball retainers also move at the average ball velocity and not with either race.
7. Ball bearings in the WAM model are modeled as single parts, not as assemblies of races, balls, retainers, shields, and lubricants.
8. Modeling most threaded holes as if only the tap drill (but not the tap) has been applied.
9. Adhesives not modeled.
10. Lubricants not modeled and not clear how to associate velocities.

Background

Historical analysis of robotic arms (Paul, Craig, Spong, etc.) assumes a set of kinematically linked rigid bodies that can be assigned coordinate frames consistent with DH parameters. The analysis holds strictly only for direct-drive robots in which the rotor is integral with one rigid body and the stator is integral with an adjacent rigid body.

While direct-drive motors have outstanding inherent backdrivability with unity transmission ratios (N=1) they are far too massive and power-inefficient for the joint-torque demands of practical robots. So virtually all conventional robots use geared speed reducers in the form of harmonic drives with transmission ratios of $40<N<400$ to match the power capability (which varies as N^2) of practical motors with the speed and torque requirements of robotic arms. DH-kinematic analysis is not equipped to account for the spinning motor rotors, which impart significant momentum effects because of the amplified speeds involved.

It has been long proposed to treat the fast-spinning backdriven motor-rotor inertias as “reflected” inertias. However, it should be said that, while the “robot math” used to calculate computed torques is precise for a direct-drive robot, even the precise calculation of reflected inertias does not and cannot account for some inertial interactions between fast spinning rotors and the velocities of the motor bodies containing those rotors. We are not aware of published analyses that have explored these errors.

U.S. Patent **Feb. 27, 1990** **Sheet 5 of 5** **4,903,536**

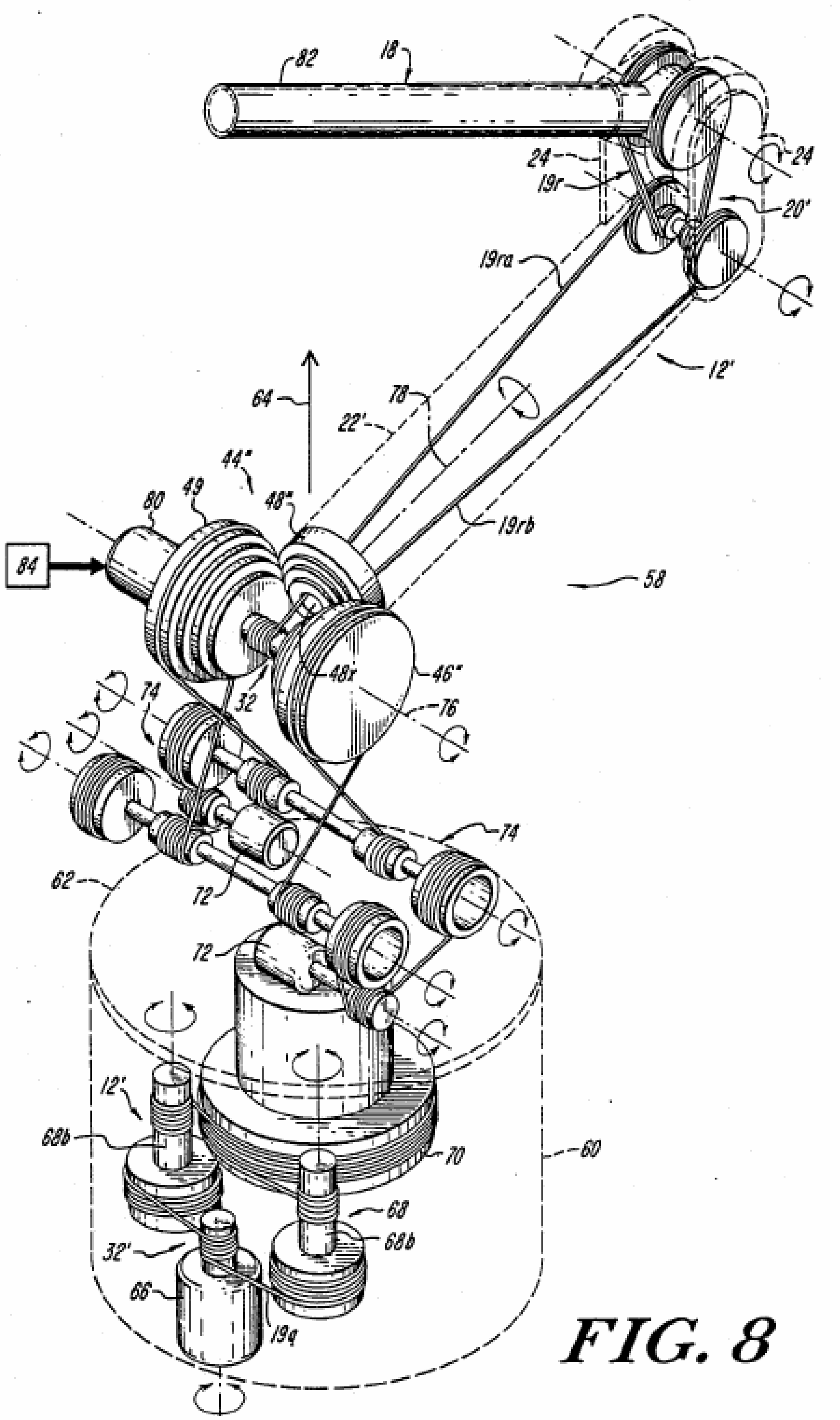

FIG. 8

Figure 1 -- Artist's sketch of WAM cable drives.

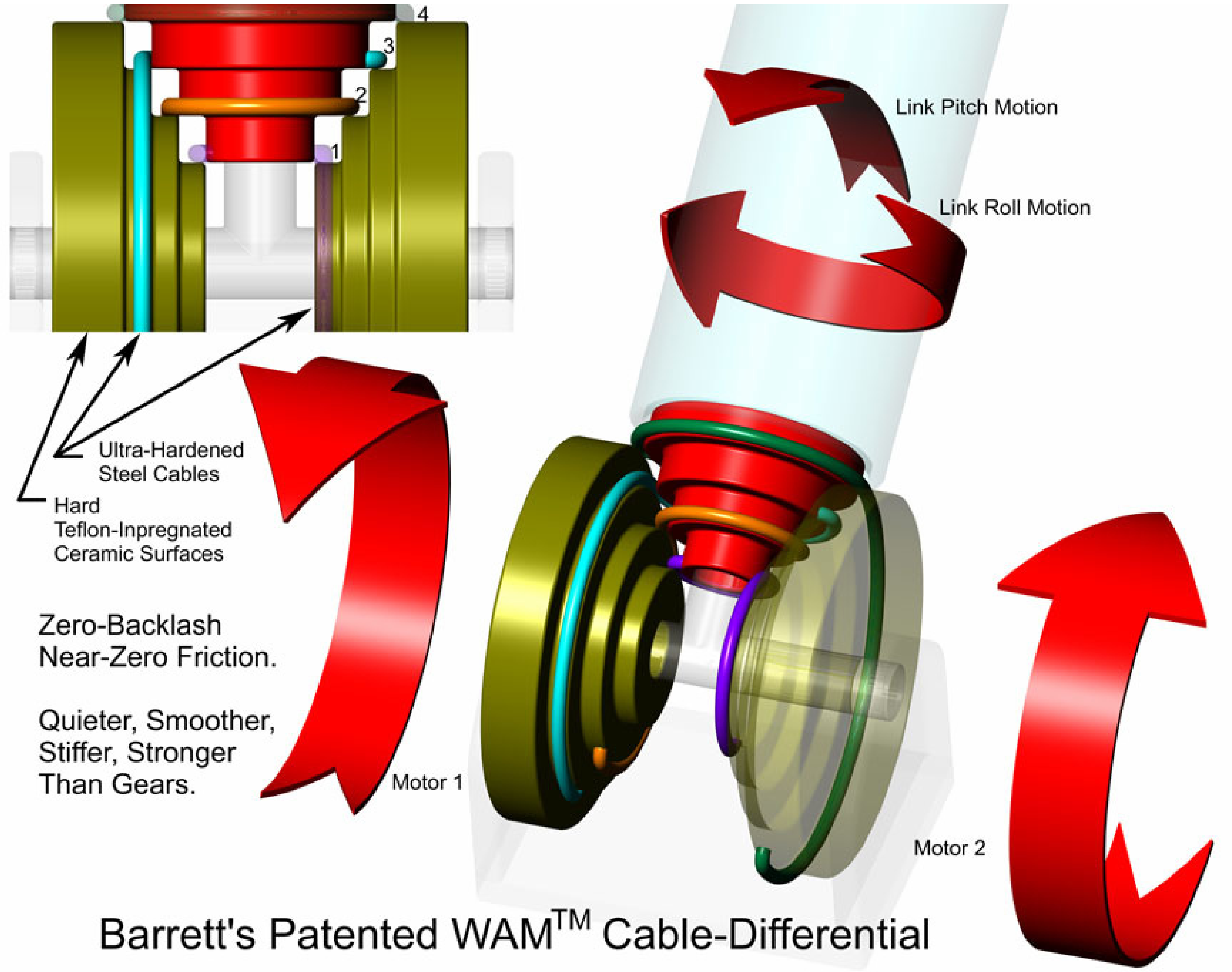

Figure 2 – Basic operation of the cabled differential mechanism used in the WAM shoulder. The WAM wrist differential operates similarly.

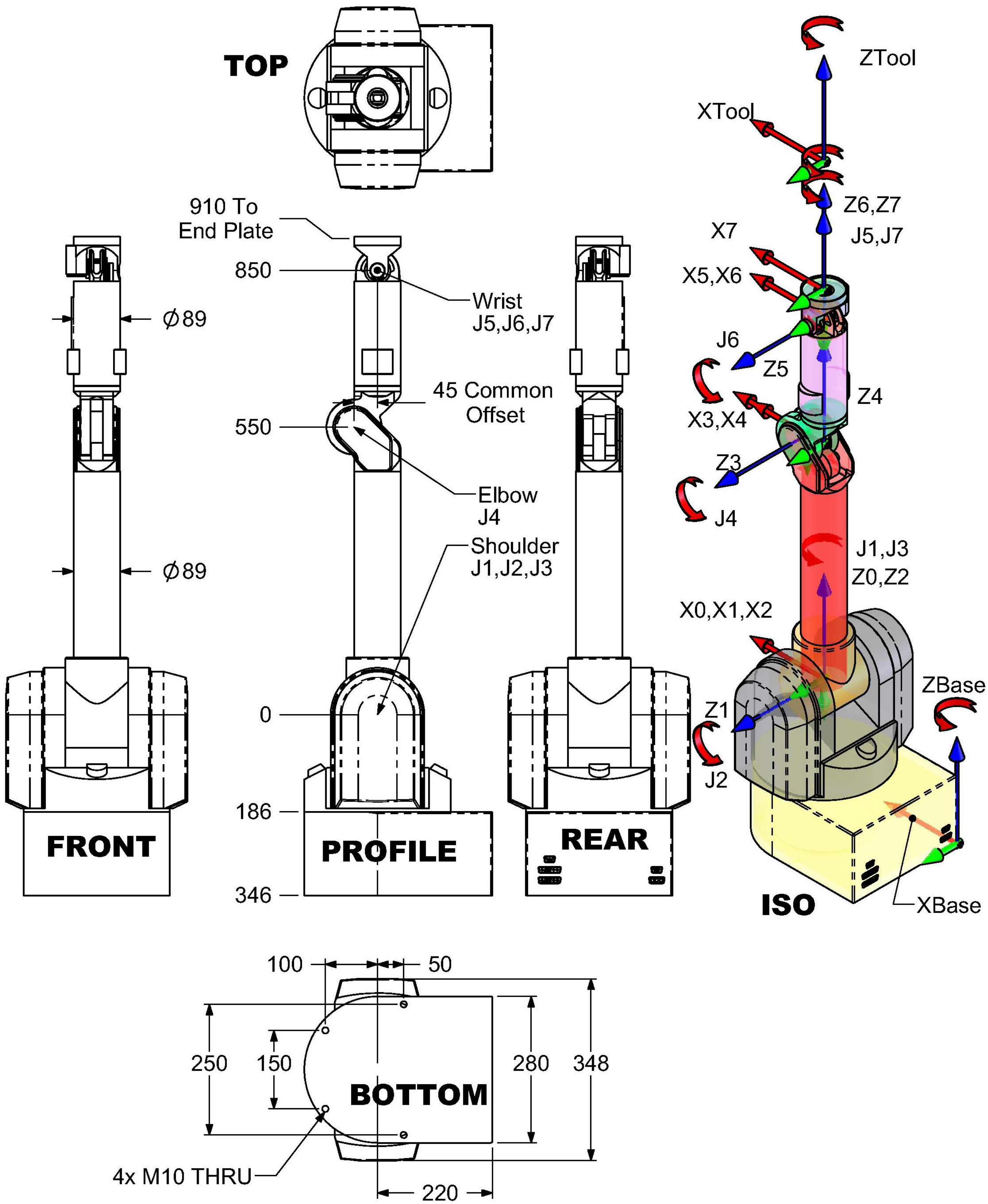

Figure 3 -- WAM Coordinate Frames.

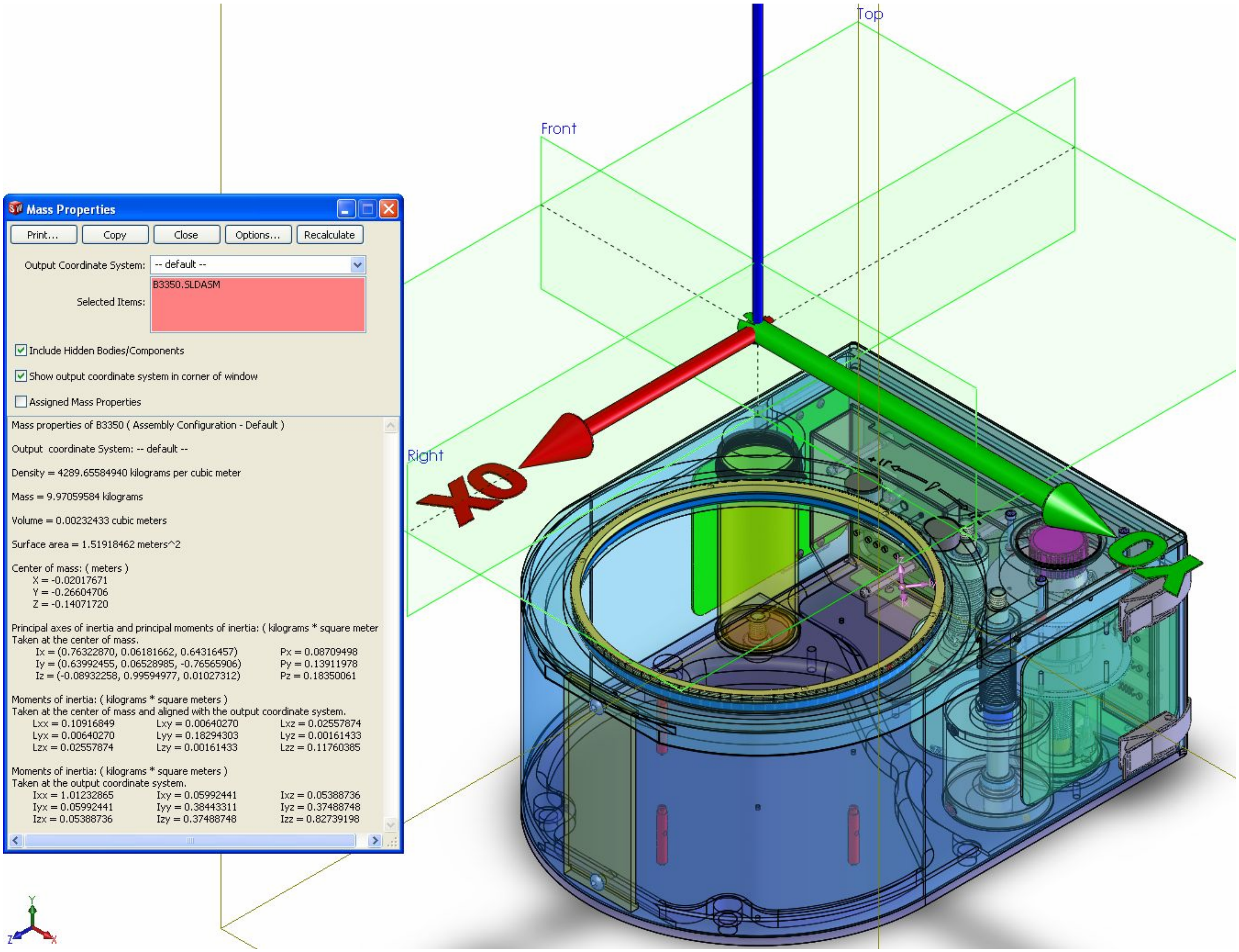

Figure 4 -- Frame 0 Inertia

Table 2 --Frame-0 Mass properties of B3350 (Assembly Configuration - Default)

Density = 4289.65584940 kilograms per cubic meter

Mass = 9.97059584 kilograms

Volume = 0.00232433 cubic meters

Surface area = 1.51918462 meters^2

Center of mass: (meters)

X = -0.02017671

Y = -0.26604706

Z = -0.14071720

Principal axes of inertia and principal moments of inertia: (kilograms * square meters)
Taken at the center of mass.

Ix = (0.76322870, 0.06181662, 0.64316457)	Px = 0.08709498
Iy = (0.63992455, 0.06528985, -0.76565906)	Py = 0.13911978
Iz = (-0.08932258, 0.99594977, 0.01027312)	Pz = 0.18350061

Moments of inertia: (kilograms * square meters)
Taken at the center of mass and aligned with the output coordinate system.

Lxx = 0.10916849	Lxy = 0.00640270	Lxz = 0.02557874
Lyx = 0.00640270	Lyy = 0.18294303	Lyz = 0.00161433
Lzx = 0.02557874	Lzy = 0.00161433	Lzz = 0.11760385

Moments of inertia: (kilograms * square meters)
Taken at the output coordinate system.

Ixx = 1.01232865	Ixy = 0.05992441	Ixz = 0.05388736
Iyx = 0.05992441	Iyy = 0.38443311	Iyz = 0.37488748
Izx = 0.05388736	Izy = 0.37488748	Izz = 0.82739198

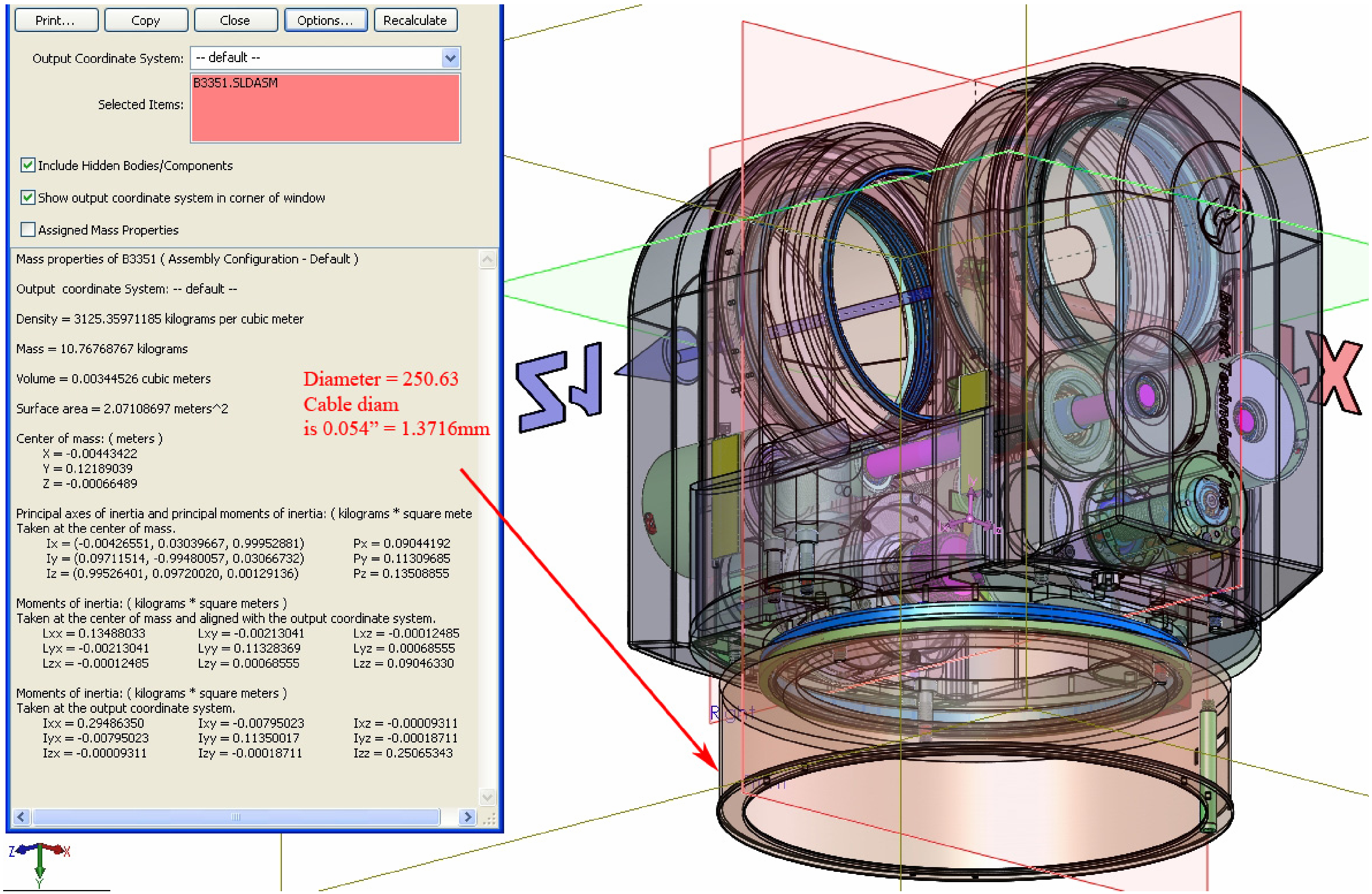

Figure 5 -- Frame-1 Inertia.

Table 3 --Frame-1 Mass properties of B3351 (Assembly Configuration - Default)

Density = 3125.35971185 kilograms per cubic meter

Mass = 10.76768767 kilograms

Volume = 0.00344526 cubic meters

Surface area = 2.07108697 meters^2

Center of mass: (meters)

X = -0.00443422

Y = 0.12189039

Z = -0.00066489

Principal axes of inertia and principal moments of inertia: (kilograms * square meters)
Taken at the center of mass.

Ix = (-0.00426551, 0.03039667, 0.99952881)	Px = 0.09044192
Iy = (0.09711514, -0.99480057, 0.03066732)	Py = 0.11309685
Iz = (0.99526401, 0.09720020, 0.00129136)	Pz = 0.13508855

Moments of inertia: (kilograms * square meters)
Taken at the center of mass and aligned with the output coordinate system.

Lxx = 0.13488033	Lxy = -0.00213041	Lxz = -0.00012485
Lyx = -0.00213041	Lyy = 0.11328369	Lyz = 0.00068555
Lzx = -0.00012485	Lzy = 0.00068555	Lzz = 0.09046330

Moments of inertia: (kilograms * square meters)
Taken at the output coordinate system.

Ixx = 0.29486350	Ixy = -0.00795023	Ixz = -0.00009311
Iyx = -0.00795023	Iyy = 0.11350017	Iyz = -0.00018711
Izx = -0.00009311	Izy = -0.00018711	Izz = 0.25065343

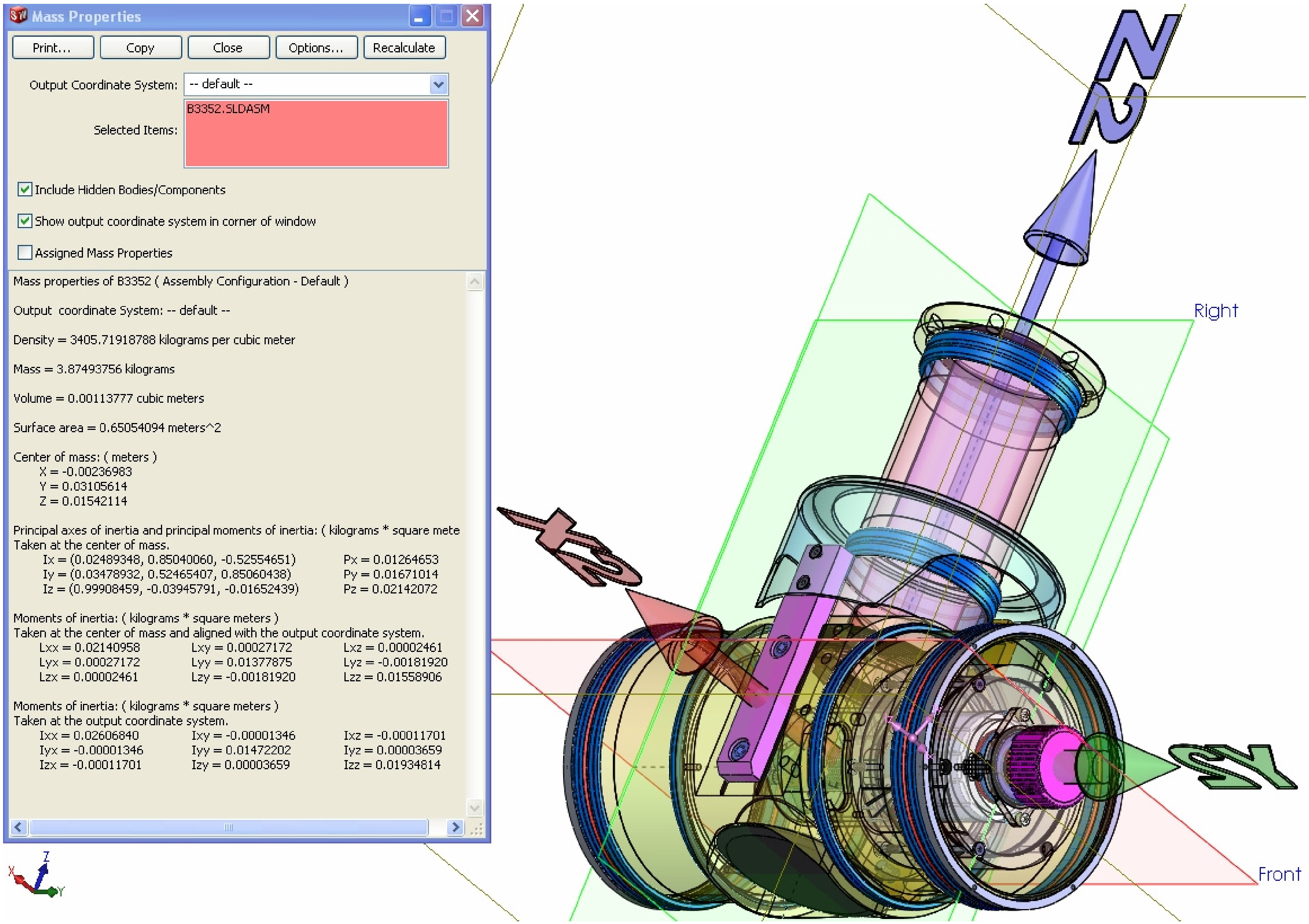

Figure 6 -- Frame-2 Inertia.

Table 4 --Frame-2 Mass properties of B3352 (Assembly Configuration - Default)

Mass = 3.87493756 kilograms

Volume = 0.00113777 cubic meters

Surface area = 0.65054094 meters^2

Center of mass: (meters)

X = -0.00236983

Y = 0.03105614

Z = 0.01542114

Principal axes of inertia and principal moments of inertia: (kilograms * square meters)

Taken at the center of mass.

Ix = (0.02489348, 0.85040060, -0.52554651)	Px = 0.01264653
Iy = (0.03478932, 0.52465407, 0.85060438)	Py = 0.01671014
Iz = (0.99908459, -0.03945791, -0.01652439)	Pz = 0.02142072

Moments of inertia: (kilograms * square meters)

Taken at the center of mass and aligned with the output coordinate system.

Lxx = 0.02140958	Lxy = 0.00027172	Lxz = 0.00002461
Lyx = 0.00027172	Lyy = 0.01377875	Lyz = -0.00181920
Lzx = 0.00002461	Lzy = -0.00181920	Lzz = 0.01558906

Moments of inertia: (kilograms * square meters)

Taken at the output coordinate system.

Ixx = 0.02606840	Ixy = -0.00001346	Ixz = -0.00011701
Iyx = -0.00001346	Iyy = 0.01472202	Iyz = 0.00003659
Izx = -0.00011701	Izy = 0.00003659	Izz = 0.01934814

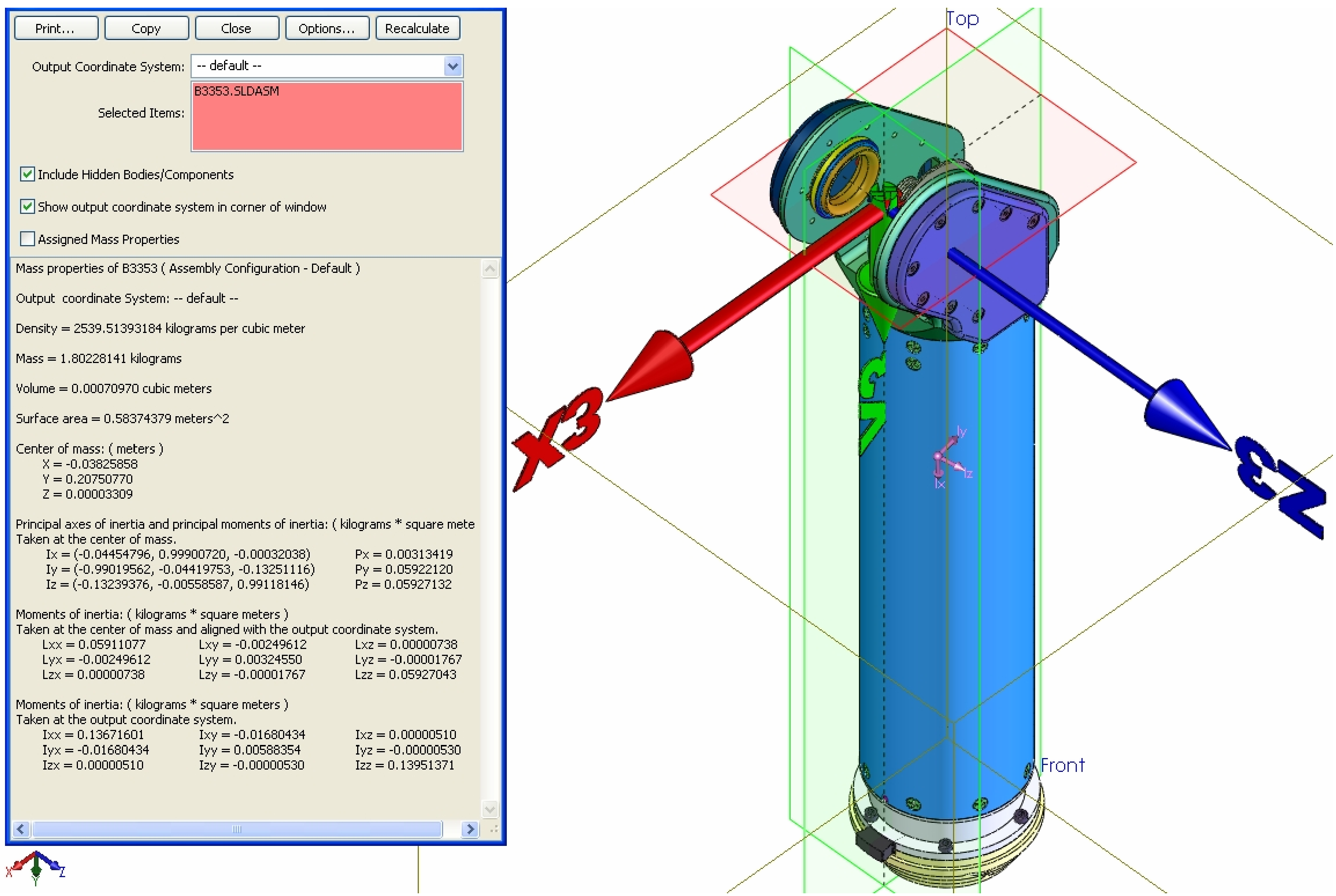

Figure 7 -- Frame-3 Inertia.

Table 5 --Frame-3 Mass properties of B3353 (Assembly Configuration - Default)

Density = 2539.51393184 kilograms per cubic meter

Mass = 1.80228141 kilograms

Volume = 0.00070970 cubic meters

Surface area = 0.58374379 meters^2

Center of mass: (meters)

X = -0.03825858

Y = 0.20750770

Z = 0.00003309

Principal axes of inertia and principal moments of inertia: (kilograms * square meters)
Taken at the center of mass.

Ix = (-0.04454796, 0.99900720, -0.00032038)	Px = 0.00313419
Iy = (-0.99019562, -0.04419753, -0.13251116)	Py = 0.05922120
Iz = (-0.13239376, -0.00558587, 0.99118146)	Pz = 0.05927132

Moments of inertia: (kilograms * square meters)
Taken at the center of mass and aligned with the output coordinate system.

Lxx = 0.05911077	Lxy = -0.00249612	Lxz = 0.00000738
Lyx = -0.00249612	Lyy = 0.00324550	Lyz = -0.00001767
Lzx = 0.00000738	Lzy = -0.00001767	Lzz = 0.05927043

Moments of inertia: (kilograms * square meters)
Taken at the output coordinate system.

Ixx = 0.13671601	Ixy = -0.01680434	Ixz = 0.00000510
Iyx = -0.01680434	Iyy = 0.00588354	Iyz = -0.00000530
Izx = 0.00000510	Izy = -0.00000530	Izz = 0.13951371

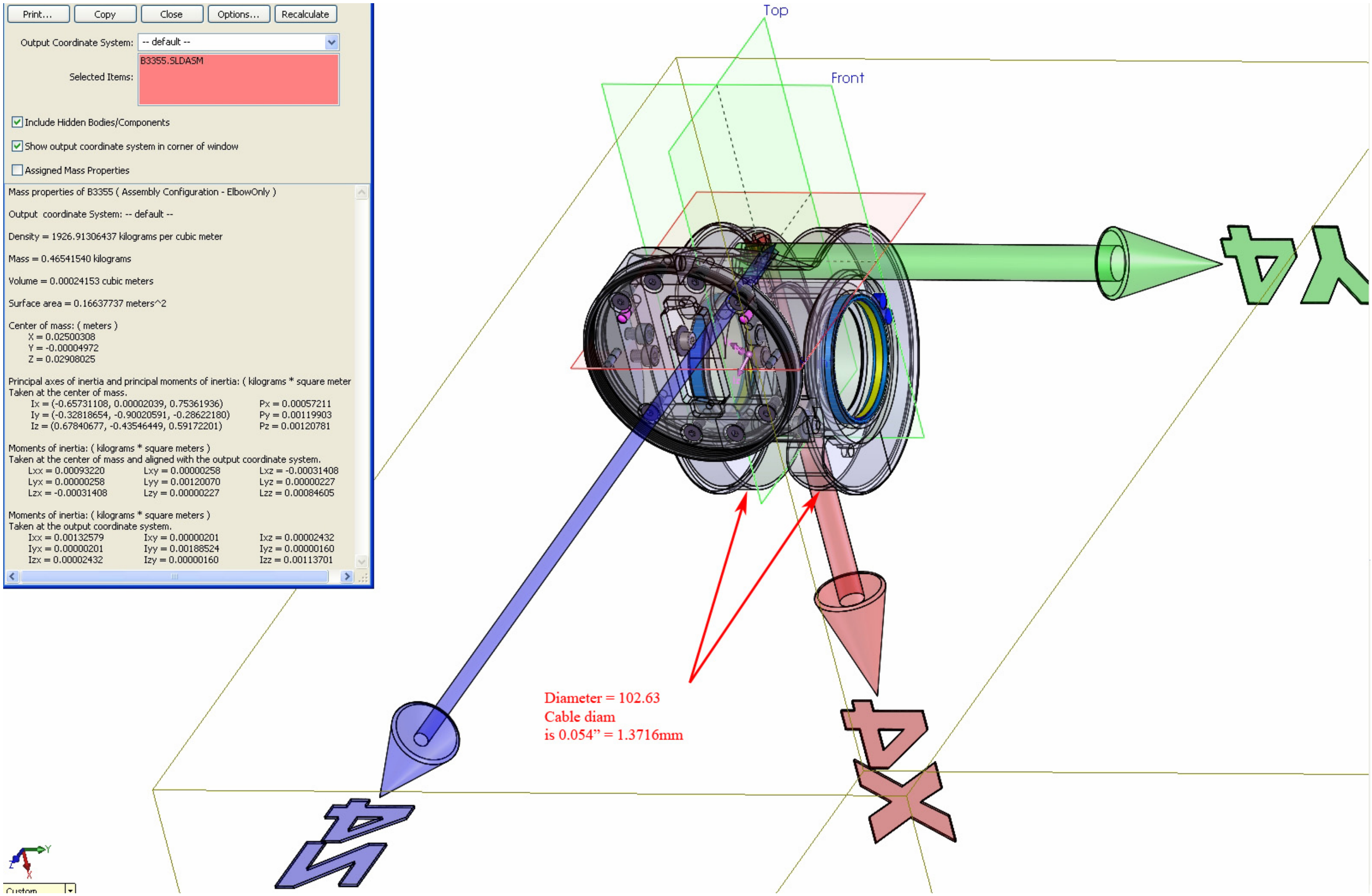

Figure 8 -- Frame-4 Inertia (without any attachment)

Table 6 --Frame-4 Mass properties of B3355 (Assembly Configuration - ElbowOnly)

Density = 1926.91306437 kilograms per cubic meter

Mass = 0.46541540 kilograms

Volume = 0.00024153 cubic meters

Surface area = 0.16637737 meters^2

Center of mass: (meters)

X = 0.02500308

Y = -0.00004972

Z = 0.02908025

Principal axes of inertia and principal moments of inertia: (kilograms * square meters)

Taken at the center of mass.

Ix = (-0.65731108, 0.00002039, 0.75361936) Px = 0.00057211

Iy = (-0.32818654, -0.90020591, -0.28622180) Py = 0.00119903

Iz = (0.67840677, -0.43546449, 0.59172201) Pz = 0.00120781

Moments of inertia: (kilograms * square meters)

Taken at the center of mass and aligned with the output coordinate system.

Lxx = 0.00093220	Lxy = 0.00000258	Lxz = -0.00031408
Lyx = 0.00000258	Lyy = 0.00120070	Lyz = 0.00000227
Lzx = -0.00031408	Lzy = 0.00000227	Lzz = 0.00084605

Moments of inertia: (kilograms * square meters)

Taken at the output coordinate system.

Ixx = 0.00132579	Ixy = 0.00000201	Ixz = 0.00002432
Iyx = 0.00000201	Iyy = 0.00188524	Iyz = 0.00000160
Izx = 0.00002432	Izy = 0.00000160	Izz = 0.00113701

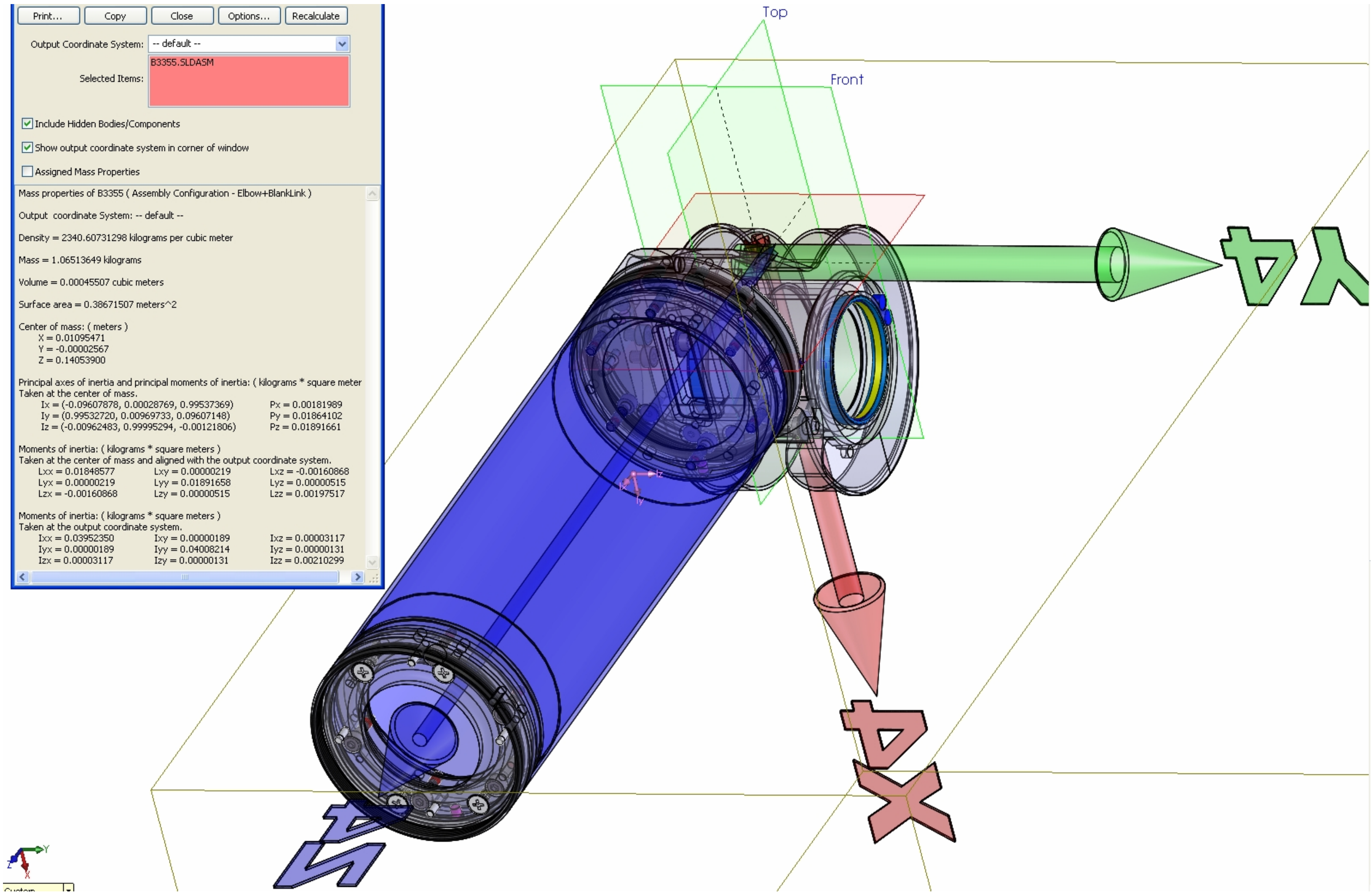

Figure 9 -- Frame-4 Inertia (with 4-DOF forearm attached).

Table 7 --Frame-4 Mass properties of B3355 (Assembly Configuration - Elbow+BlankLink)

Density = 2340.60731298 kilograms per cubic meter

Mass = 1.06513649 kilograms

Volume = 0.00045507 cubic meters

Surface area = 0.38671507 meters^2

Center of mass: (meters)

X = 0.01095471

Y = -0.00002567

Z = 0.14053900

Principal axes of inertia and principal moments of inertia: (kilograms * square meters)

Taken at the center of mass.

Ix = (-0.09607878, 0.00028769, 0.99537369)	Px = 0.00181989
Iy = (0.99532720, 0.00969733, 0.09607148)	Py = 0.01864102
Iz = (-0.00962483, 0.99995294, -0.00121806)	Pz = 0.01891661

Moments of inertia: (kilograms * square meters)

Taken at the center of mass and aligned with the output coordinate system.

Lxx = 0.01848577	Lxy = 0.00000219	Lxz = -0.00160868
Lyx = 0.00000219	Lyy = 0.01891658	Lyz = 0.00000515
Lzx = -0.00160868	Lzy = 0.00000515	Lzz = 0.00197517

Moments of inertia: (kilograms * square meters)

Taken at the output coordinate system.

Ixx = 0.03952350	Ixy = 0.00000189	Ixz = 0.00003117
Iyx = 0.00000189	Iyy = 0.04008214	Iyz = 0.00000131
Izx = 0.00003117	Izy = 0.00000131	Izz = 0.00210299

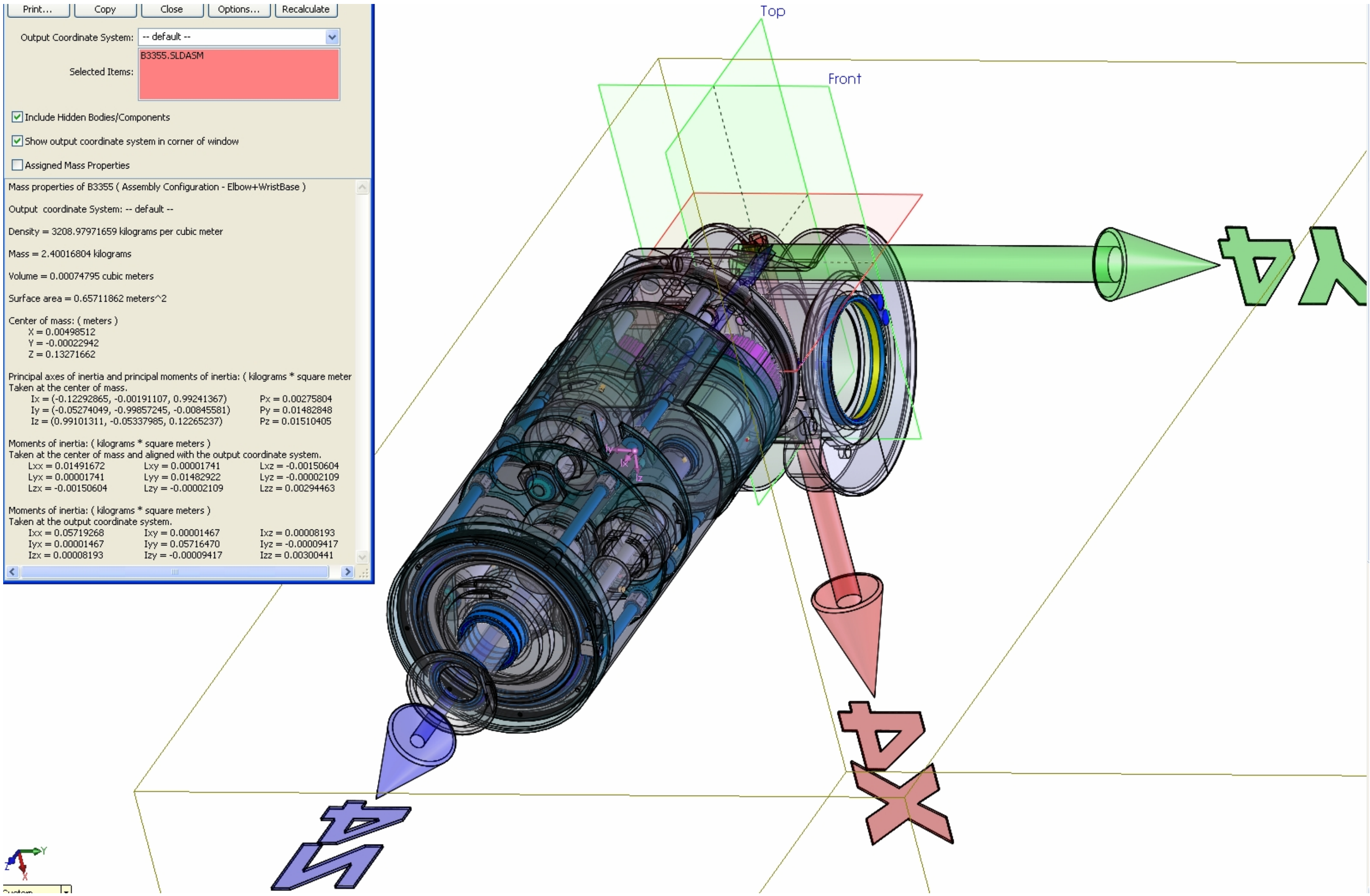

Figure 10 -- Frame-4 Inertia (with base of wrist attached).

Table 8 --Frame-4 Mass properties of B3355 (Assembly Configuration - Elbow+WristBase)

Density = 3208.97971659 kilograms per cubic meter

Mass = 2.40016804 kilograms

Volume = 0.00074795 cubic meters

Surface area = 0.65711862 meters^2

Center of mass: (meters)

X = 0.00498512

Y = -0.00022942

Z = 0.13271662

Principal axes of inertia and principal moments of inertia: (kilograms * square meters)

Taken at the center of mass.

Ix = (-0.12292865, -0.00191107, 0.99241367)	Px = 0.00275804
Iy = (-0.05274049, -0.99857245, -0.00845581)	Py = 0.01482848
Iz = (0.99101311, -0.05337985, 0.12265237)	Pz = 0.01510405

Moments of inertia: (kilograms * square meters)

Taken at the center of mass and aligned with the output coordinate system.

Lxx = 0.01491672	Lxy = 0.00001741	Lxz = -0.00150604
Lyx = 0.00001741	Lyy = 0.01482922	Lyz = -0.00002109
Lzx = -0.00150604	Lzy = -0.00002109	Lzz = 0.00294463

Moments of inertia: (kilograms * square meters)

Taken at the output coordinate system.

Ixx = 0.05719268	Ixy = 0.00001467	Ixz = 0.00008193
Iyx = 0.00001467	Iyy = 0.05716470	Iyz = -0.00009417
Izx = 0.00008193	Izy = -0.00009417	Izz = 0.00300441

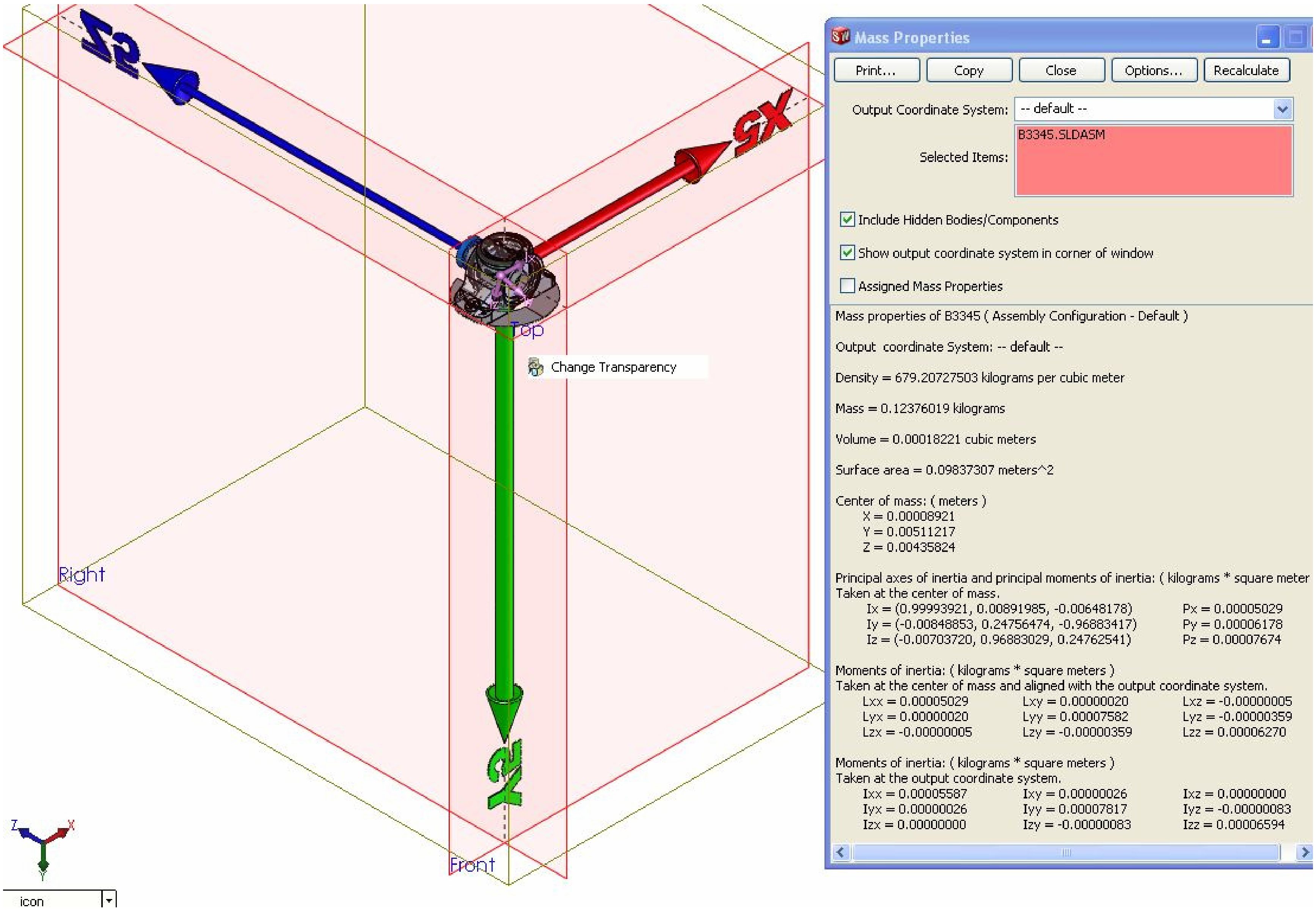

Figure 11 – Frame-5 Inertia

Table 9 --Frame-5 Mass properties of B3345 (Assembly Configuration - Default)

Density = 679.20727503 kilograms per cubic meter

Mass = 0.12376019 kilograms

Volume = 0.00018221 cubic meters

Surface area = 0.09837307 meters^2

Center of mass: (meters)

X = 0.00008921

Y = 0.00511217

Z = 0.00435824

Principal axes of inertia and principal moments of inertia: (kilograms * square meters)

Taken at the center of mass.

Ix = (0.99993921, 0.00891985, -0.00648178)	Px = 0.00005029
Iy = (-0.00848853, 0.24756474, -0.96883417)	Py = 0.00006178
Iz = (-0.00703720, 0.96883029, 0.24762541)	Pz = 0.00007674

Moments of inertia: (kilograms * square meters)

Taken at the center of mass and aligned with the output coordinate system.

Lxx = 0.00005029	Lxy = 0.00000020	Lxz = -0.00000005
Lyx = 0.00000020	Lyy = 0.00007582	Lyz = -0.00000359
Lzx = -0.00000005	Lzy = -0.00000359	Lzz = 0.00006270

Moments of inertia: (kilograms * square meters)

Taken at the output coordinate system.

Ixx = 0.00005587	Ixy = 0.00000026	Ixz = 0.00000000
Iyx = 0.00000026	Iyy = 0.00007817	Iyz = -0.00000083
Izx = 0.00000000	Izy = -0.00000083	Izz = 0.00006594

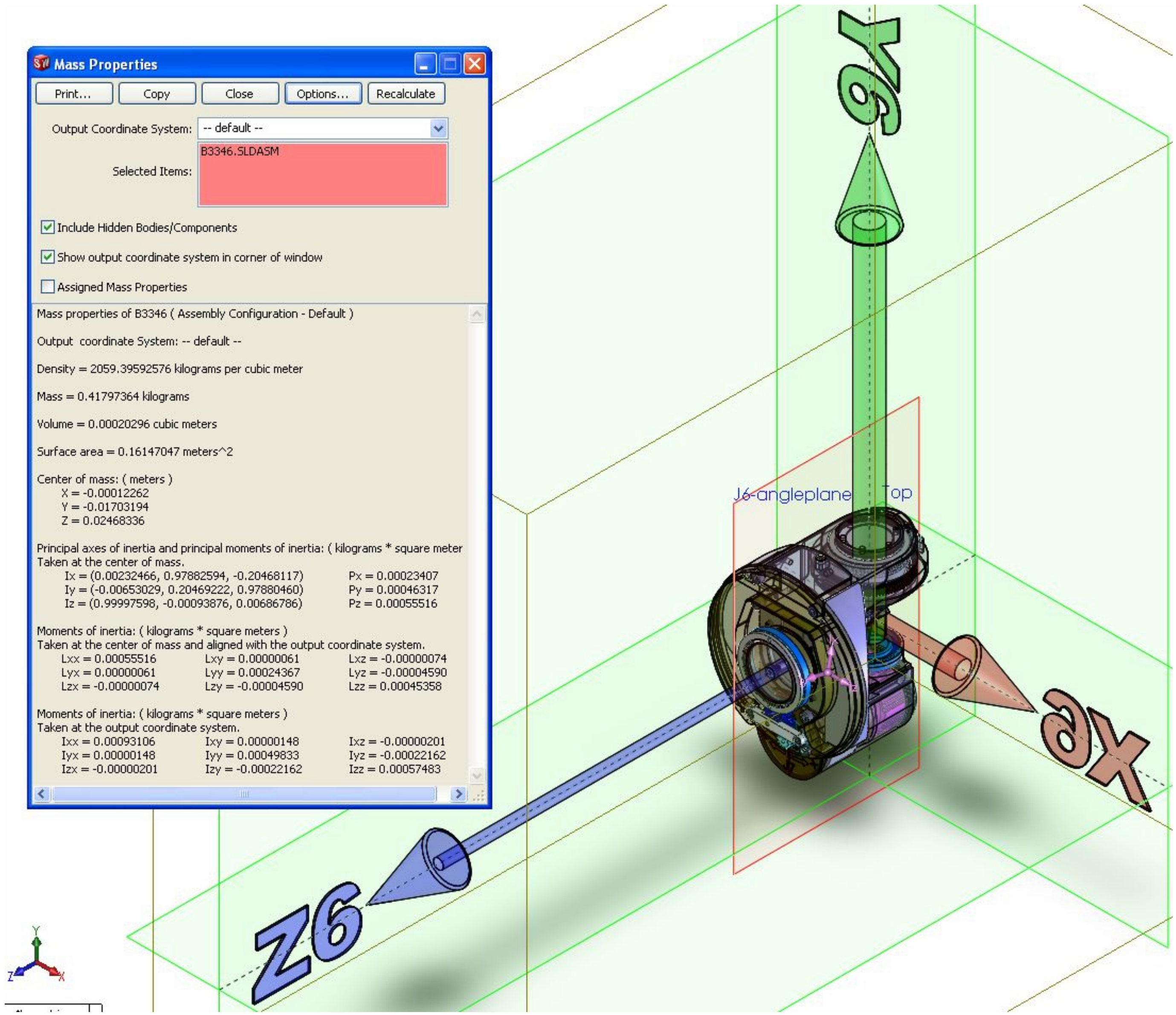

Figure 12 -- Frame-6 Inertia.

Table 10 --Frame-6 Mass properties of B3346 (Assembly Configuration - Default)

Density = 2059.39592576 kilograms per cubic meter

Mass = 0.41797364 kilograms

Volume = 0.00020296 cubic meters

Surface area = 0.16147047 meters^2

Center of mass: (meters)

X = -0.00012262

Y = -0.01703194

Z = 0.02468336

Principal axes of inertia and principal moments of inertia: (kilograms * square meters)

Taken at the center of mass.

Ix = (0.00232466, 0.97882594, -0.20468117)	Px = 0.00023407
Iy = (-0.00653029, 0.20469222, 0.97880460)	Py = 0.00046317
Iz = (0.99997598, -0.00093876, 0.00686786)	Pz = 0.00055516

Moments of inertia: (kilograms * square meters)

Taken at the center of mass and aligned with the output coordinate system.

Lxx = 0.00055516	Lxy = 0.00000061	Lxz = -0.00000074
Lyx = 0.00000061	Lyy = 0.00024367	Lyz = -0.00004590
Lzx = -0.00000074	Lzy = -0.00004590	Lzz = 0.00045358

Moments of inertia: (kilograms * square meters)

Taken at the output coordinate system.

Ixx = 0.00093106	Ixy = 0.00000148	Ixz = -0.00000201
Iyx = 0.00000148	Iyy = 0.00049833	Iyz = -0.00022162
Izx = -0.00000201	Izy = -0.00022162	Izz = 0.00057483

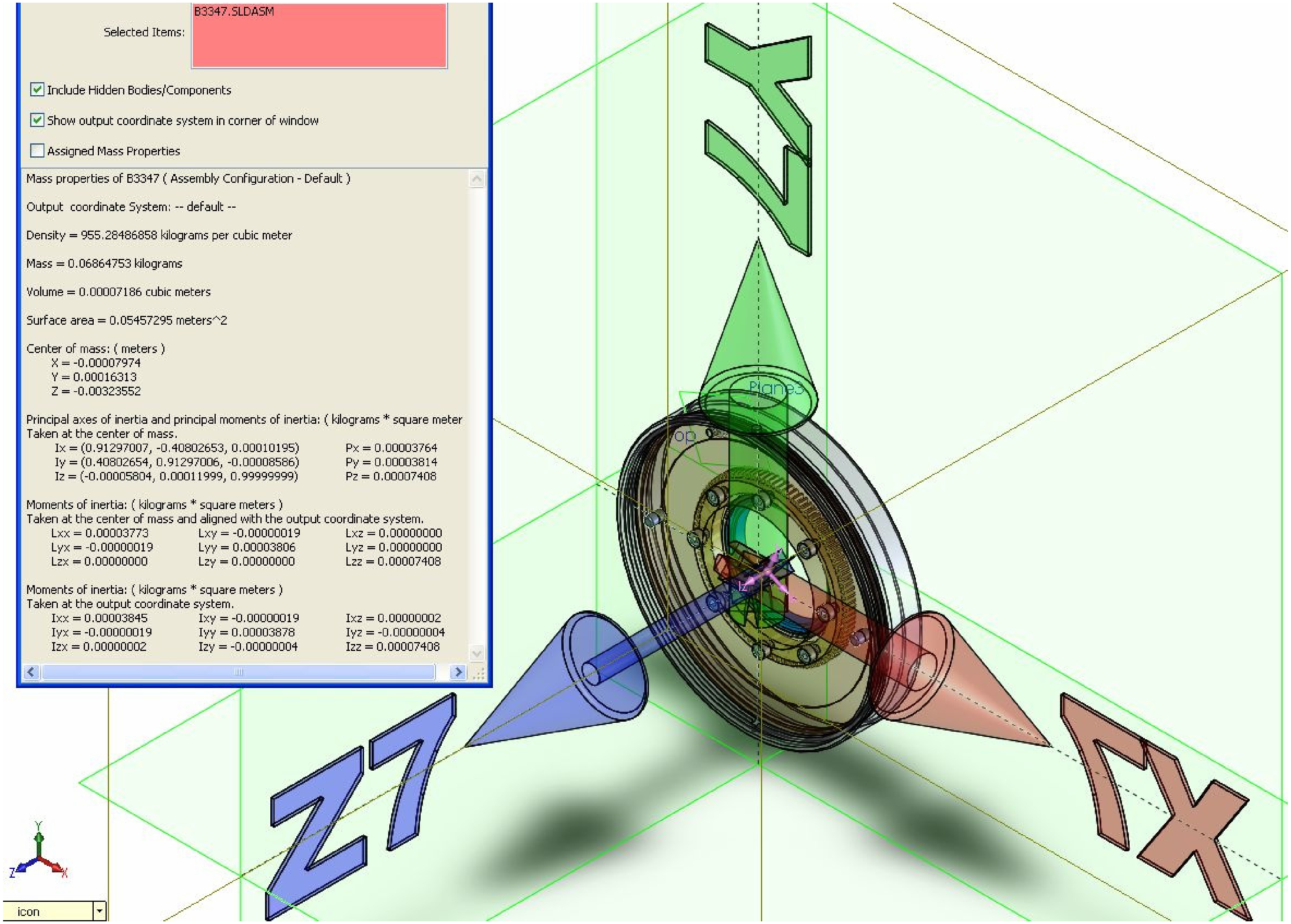

Figure 13 -- Frame-7 Inertia.

Table 11 --Frame-7 Mass properties of B3347 (Assembly Configuration - Default)

Density = 955.28486858 kilograms per cubic meter

Mass = 0.06864753 kilograms

Volume = 0.00007186 cubic meters

Surface area = 0.05457295 meters^2

Center of mass: (meters)

X = -0.00007974

Y = 0.00016313

Z = -0.00323552

Principal axes of inertia and principal moments of inertia: (kilograms * square meters)
Taken at the center of mass.

Ix = (0.91297007, -0.40802653, 0.00010195)	Px = 0.00003764
Iy = (0.40802654, 0.91297006, -0.00008586)	Py = 0.00003814
Iz = (-0.00005804, 0.00011999, 0.99999999)	Pz = 0.00007408

Moments of inertia: (kilograms * square meters)
Taken at the center of mass and aligned with the output coordinate system.

Lxx = 0.00003773	Lxy = -0.00000019	Lxz = 0.00000000
Lyx = -0.00000019	Lyy = 0.00003806	Lyz = 0.00000000
Lzx = 0.00000000	Lzy = 0.00000000	Lzz = 0.00007408

Moments of inertia: (kilograms * square meters)
Taken at the output coordinate system.

Ixx = 0.00003845	Ixy = -0.00000019	Ixz = 0.00000002
Iyx = -0.00000019	Iyy = 0.00003878	Iyz = -0.00000004
Izx = 0.00000002	Izy = -0.00000004	Izz = 0.00007408

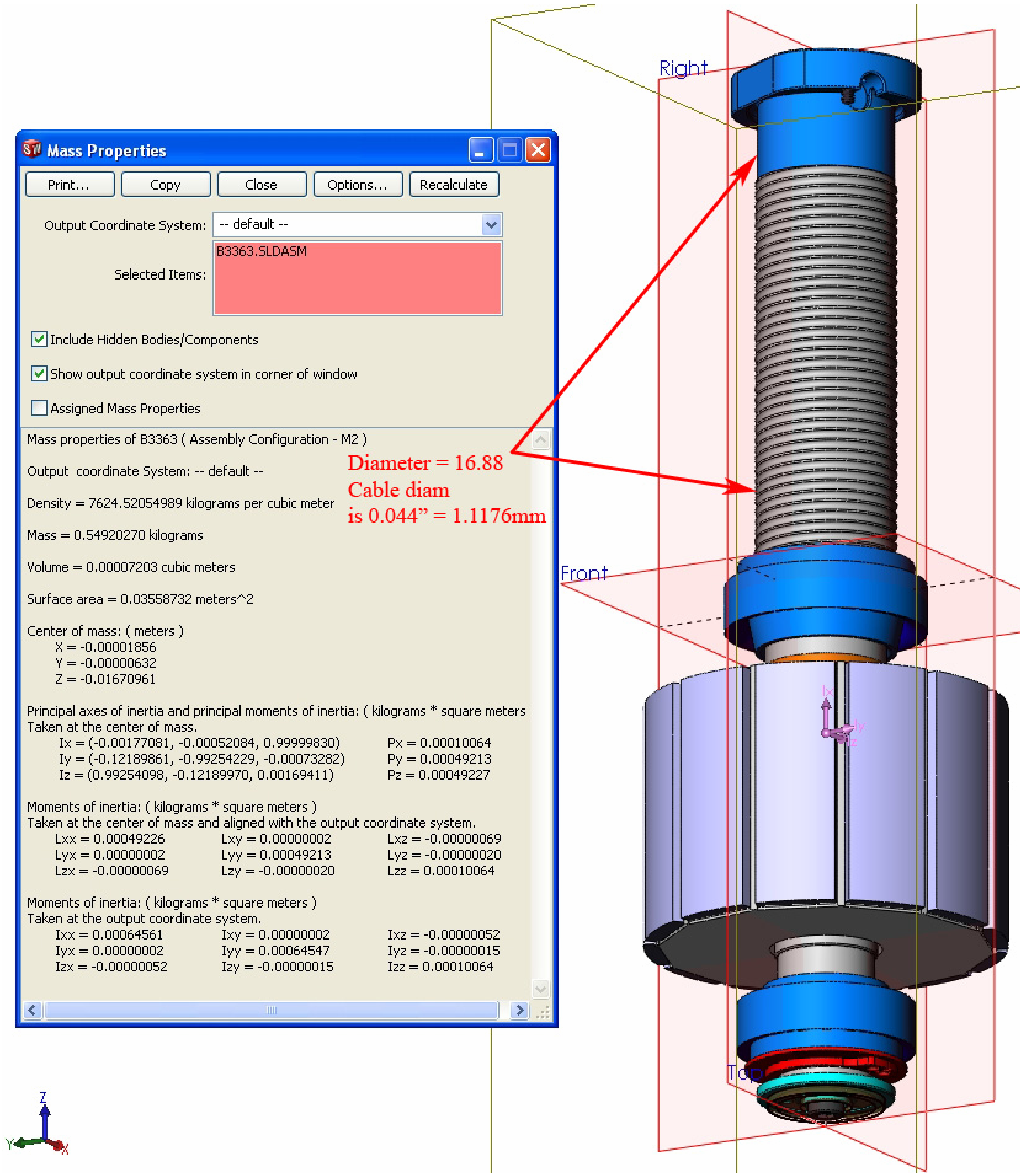

Figure 14 -- Motor Rotor for the 1st Four DOFs of the WAM.

Table 12 --Mass properties of B3363 (Assembly Configuration - M2)

Density = 7624.52054989 kilograms per cubic meter

Mass = 0.54920270 kilograms

Volume = 0.00007203 cubic meters

Surface area = 0.03558732 meters^2

Center of mass: (meters)

X = -0.00001856

Y = -0.00000632

Z = -0.01670961

Principal axes of inertia and principal moments of inertia: (kilograms * square meters)
Taken at the center of mass.

Ix = (-0.00177081, -0.00052084, 0.99999830)	Px = 0.00010064
Iy = (-0.12189861, -0.99254229, -0.00073282)	Py = 0.00049213
Iz = (0.99254098, -0.12189970, 0.00169411)	Pz = 0.00049227

Moments of inertia: (kilograms * square meters)
Taken at the center of mass and aligned with the output coordinate system.

Lxx = 0.00049226	Lxy = 0.00000002	Lxz = -0.00000069
Lyx = 0.00000002	Lyy = 0.00049213	Lyz = -0.00000020
Lzx = -0.00000069	Lzy = -0.00000020	Lzz = 0.00010064

Moments of inertia: (kilograms * square meters)
Taken at the output coordinate system.

Ixx = 0.00064561	Ixy = 0.00000002	Ixz = -0.00000052
Iyx = 0.00000002	Iyy = 0.00064547	Iyz = -0.00000015
Izx = -0.00000052	Izy = -0.00000015	**Izz = 0.00010064**

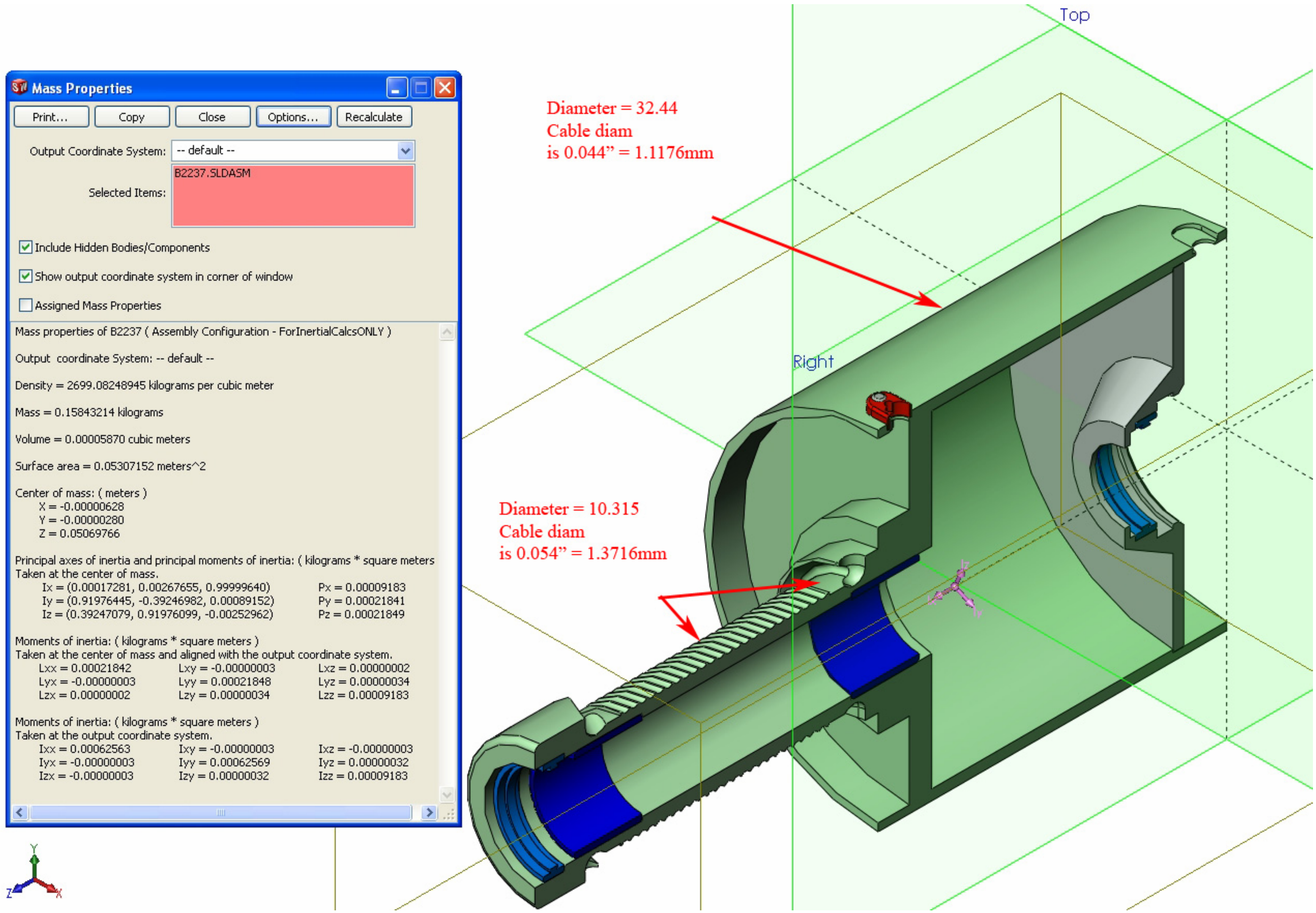

Figure 15 -- "Grenade" that couples 1st and 2nd stages of the 1st three DOFs of the WAM.

Table 13 --Mass properties of B2237 (Assembly Configuration - ForInertialCalcsONLY)

Density = 2699.08248945 kilograms per cubic meter

Mass = 0.15843214 kilograms

Volume = 0.00005870 cubic meters

Surface area = 0.05307152 meters^2

Center of mass: (meters)

X = -0.00000628

Y = -0.00000280

Z = 0.05069766

Principal axes of inertia and principal moments of inertia: (kilograms * square meters)

Taken at the center of mass.

Ix = (0.00017281, 0.00267655, 0.99999640)	Px = 0.00009183
Iy = (0.91976445, -0.39246982, 0.00089152)	Py = 0.00021841
Iz = (0.39247079, 0.91976099, -0.00252962)	Pz = 0.00021849

Moments of inertia: (kilograms * square meters)

Taken at the center of mass and aligned with the output coordinate system.

Lxx = 0.00021842	Lxy = -0.00000003	Lxz = 0.00000002
Lyx = -0.00000003	Lyy = 0.00021848	Lyz = 0.00000034
Lzx = 0.00000002	Lzy = 0.00000034	Lzz = 0.00009183

Moments of inertia: (kilograms * square meters)

Taken at the output coordinate system.

Ixx = 0.00062563	Ixy = -0.00000003	Ixz = -0.00000003
Iyx = -0.00000003	Iyy = 0.00062569	Iyz = 0.00000032
Izx = -0.00000003	Izy = 0.00000032	**Izz = 0.00009183**

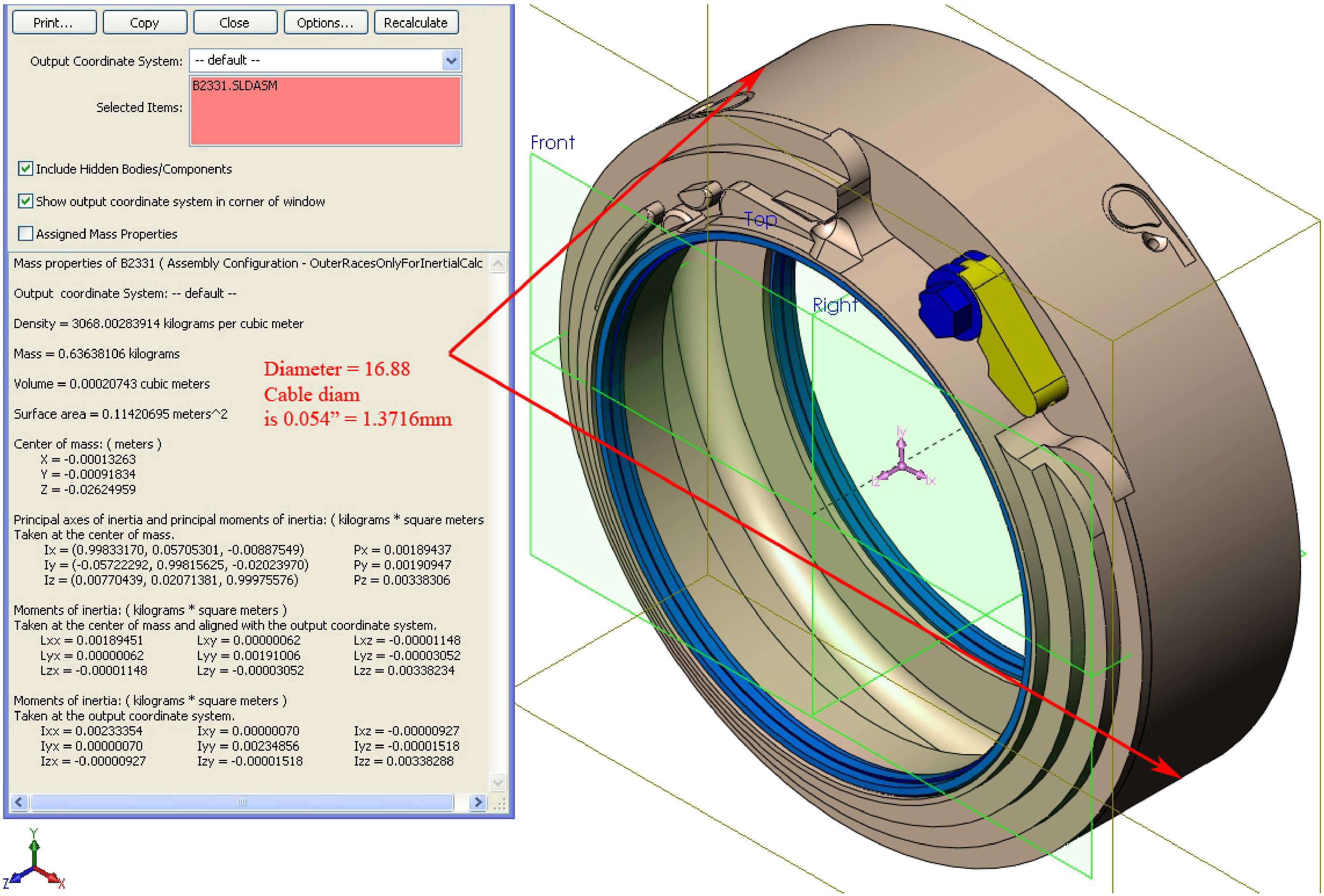

Figure 16 -- Differential-Input Pulley.

Table 14 --Mass properties of B2331 (Assembly Configuration - OuterRacesOnlyForInertialCalcs)

Density = 3068.00283914 kilograms per cubic meter

Mass = 0.63638106 kilograms

Volume = 0.00020743 cubic meters

Surface area = 0.11420695 meters^2

Center of mass: (meters)

X = -0.00013263

Y = -0.00091834

Z = -0.02624959

Principal axes of inertia and principal moments of inertia: (kilograms * square meters)

Taken at the center of mass.

Ix = (0.99833170, 0.05705301, -0.00887549)	Px = 0.00189437
Iy = (-0.05722292, 0.99815625, -0.02023970)	Py = 0.00190947
Iz = (0.00770439, 0.02071381, 0.99975576)	Pz = 0.00338306

Moments of inertia: (kilograms * square meters)

Taken at the center of mass and aligned with the output coordinate system.

Lxx = 0.00189451	Lxy = 0.00000062	Lxz = -0.00001148
Lyx = 0.00000062	Lyy = 0.00191006	Lyz = -0.00003052
Lzx = -0.00001148	Lzy = -0.00003052	Lzz = 0.00338234

Moments of inertia: (kilograms * square meters)

Taken at the output coordinate system.

Ixx = 0.00233354	Ixy = 0.00000070	Ixz = -0.00000927
Iyx = 0.00000070	Iyy = 0.00234856	Iyz = -0.00001518
Izx = -0.00000927	Izy = -0.00001518	**Izz = 0.00338288**

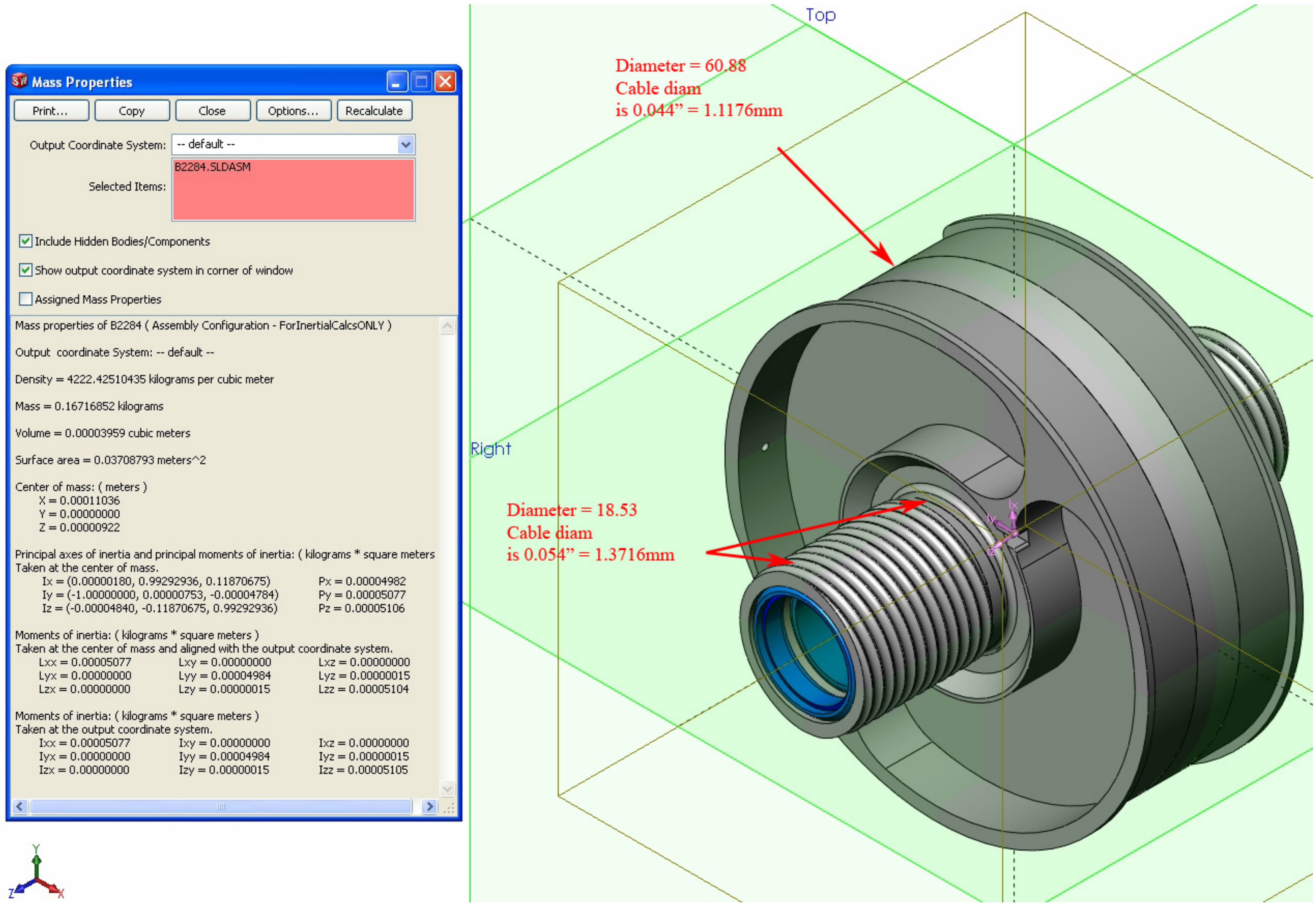

Figure 17 -- Elbow Pulley-Pinion with all bearings that couples the 1st & 2nd stages of J4.

Table 15 --Mass properties of B2284 (Assembly Configuration - ForInertialCalcsONLY)

Density = 4222.42510435 kilograms per cubic meter

Mass = 0.16716852 kilograms

Volume = 0.00003959 cubic meters

Surface area = 0.03708793 meters^2

Center of mass: (meters)
X = 0.00011036
Y = 0.00000000
Z = 0.00000922

Principal axes of inertia and principal moments of inertia: (kilograms * square meters)
Taken at the center of mass.

Ix = (0.00000180, 0.99292936, 0.11870675)	Px = 0.00004982
Iy = (-1.00000000, 0.00000753, -0.00004784)	Py = 0.00005077
Iz = (-0.00004840, -0.11870675, 0.99292936)	Pz = 0.00005106

Moments of inertia: (kilograms * square meters)
Taken at the center of mass and aligned with the output coordinate system.

Lxx = 0.00005077	Lxy = 0.00000000	Lxz = 0.00000000
Lyx = 0.00000000	Lyy = 0.00004984	Lyz = 0.00000015
Lzx = 0.00000000	Lzy = 0.00000015	Lzz = 0.00005104

Moments of inertia: (kilograms * square meters)
Taken at the output coordinate system.

Ixx = 0.00005077	Ixy = 0.00000000	Ixz = 0.00000000
Iyx = 0.00000000	Iyy = 0.00004984	Iyz = 0.00000015
Izx = 0.00000000	Izy = 0.00000015	**Izz = 0.00005105**

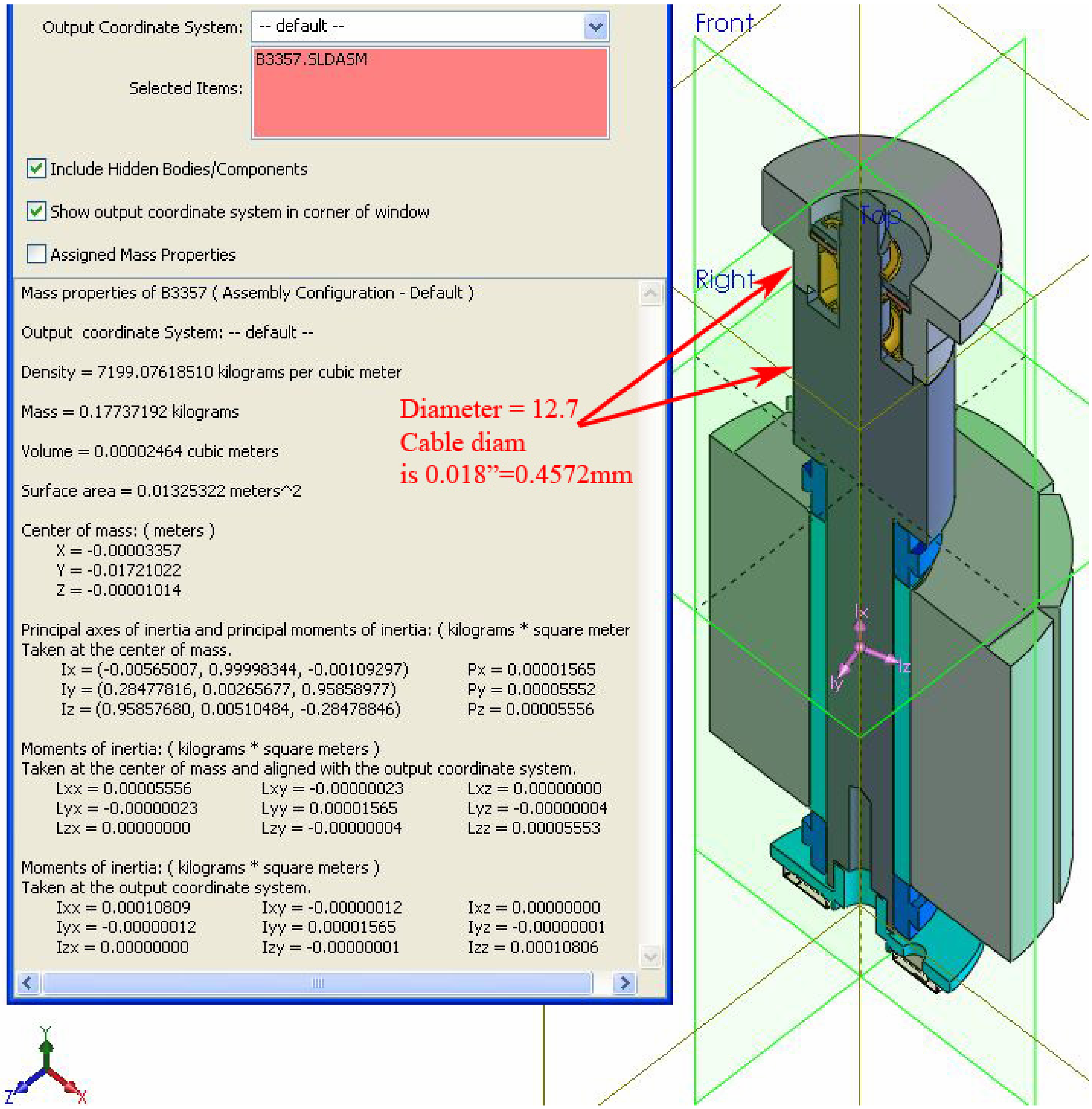

Figure 18 -- M5/M6 Motor Inertia.

Table 16 --Mass properties of B3357 (Assembly Configuration - Default)

Density = 7199.07618510 kilograms per cubic meter

Mass = 0.17737192 kilograms

Volume = 0.00002464 cubic meters

Surface area = 0.01325322 meters^2

Center of mass: (meters)

X = -0.00003357

Y = -0.01721022

Z = -0.00001014

Principal axes of inertia and principal moments of inertia: (kilograms * square meters)

Taken at the center of mass.

Ix = (-0.00565007, 0.99998344, -0.00109297)	Px = 0.00001565
Iy = (0.28477816, 0.00265677, 0.95858977)	Py = 0.00005552
Iz = (0.95857680, 0.00510484, -0.28478846)	Pz = 0.00005556

Moments of inertia: (kilograms * square meters)

Taken at the center of mass and aligned with the output coordinate system.

Lxx = 0.00005556	Lxy = -0.00000023	Lxz = 0.00000000
Lyx = -0.00000023	Lyy = 0.00001565	Lyz = -0.00000004
Lzx = 0.00000000	Lzy = -0.00000004	Lzz = 0.00005553

Moments of inertia: (kilograms * square meters)

Taken at the output coordinate system.

Ixx = 0.00010809	Ixy = -0.00000012	Ixz = 0.00000000
Iyx = -0.00000012	**Iyy = 0.00001565**	Iyz = -0.00000001
Izx = 0.00000000	Izy = -0.00000001	Izz = 0.00010806

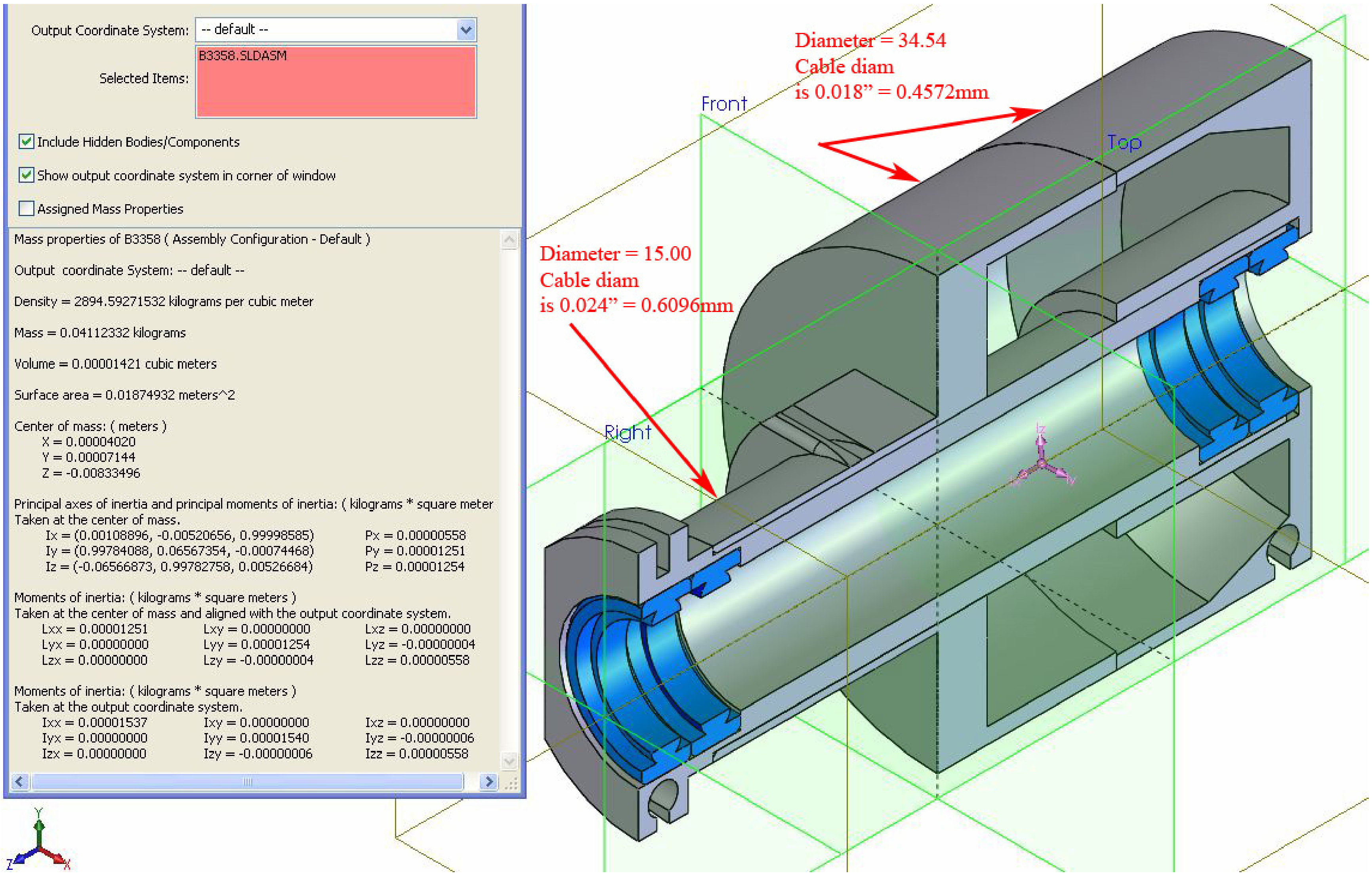

Figure 19 -- Stage-1-2 PulleyPinion.

Table 17 --Mass properties of B3358 (Assembly Configuration - Default)

Density = 2894.59271532 kilograms per cubic meter

Mass = 0.04112332 kilograms

Volume = 0.00001421 cubic meters

Surface area = 0.01874932 meters^2

Center of mass: (meters)

X = 0.00004020

Y = 0.00007144

Z = -0.00833496

Principal axes of inertia and principal moments of inertia: (kilograms * square meters)

Taken at the center of mass.

Ix = (0.00108896, -0.00520656, 0.99998585)	Px = 0.00000558
Iy = (0.99784088, 0.06567354, -0.00074468)	Py = 0.00001251
Iz = (-0.06566873, 0.99782758, 0.00526684)	Pz = 0.00001254

Moments of inertia: (kilograms * square meters)

Taken at the center of mass and aligned with the output coordinate system.

Lxx = 0.00001251	Lxy = 0.00000000	Lxz = 0.00000000
Lyx = 0.00000000	Lyy = 0.00001254	Lyz = -0.00000004
Lzx = 0.00000000	Lzy = -0.00000004	Lzz = 0.00000558

Moments of inertia: (kilograms * square meters)

Taken at the output coordinate system.

Ixx = 0.00001537	Ixy = 0.00000000	Ixz = 0.00000000
Iyx = 0.00000000	Iyy = 0.00001540	Iyz = -0.00000006
Izx = 0.00000000	Izy = -0.00000006	**Izz = 0.00000558**

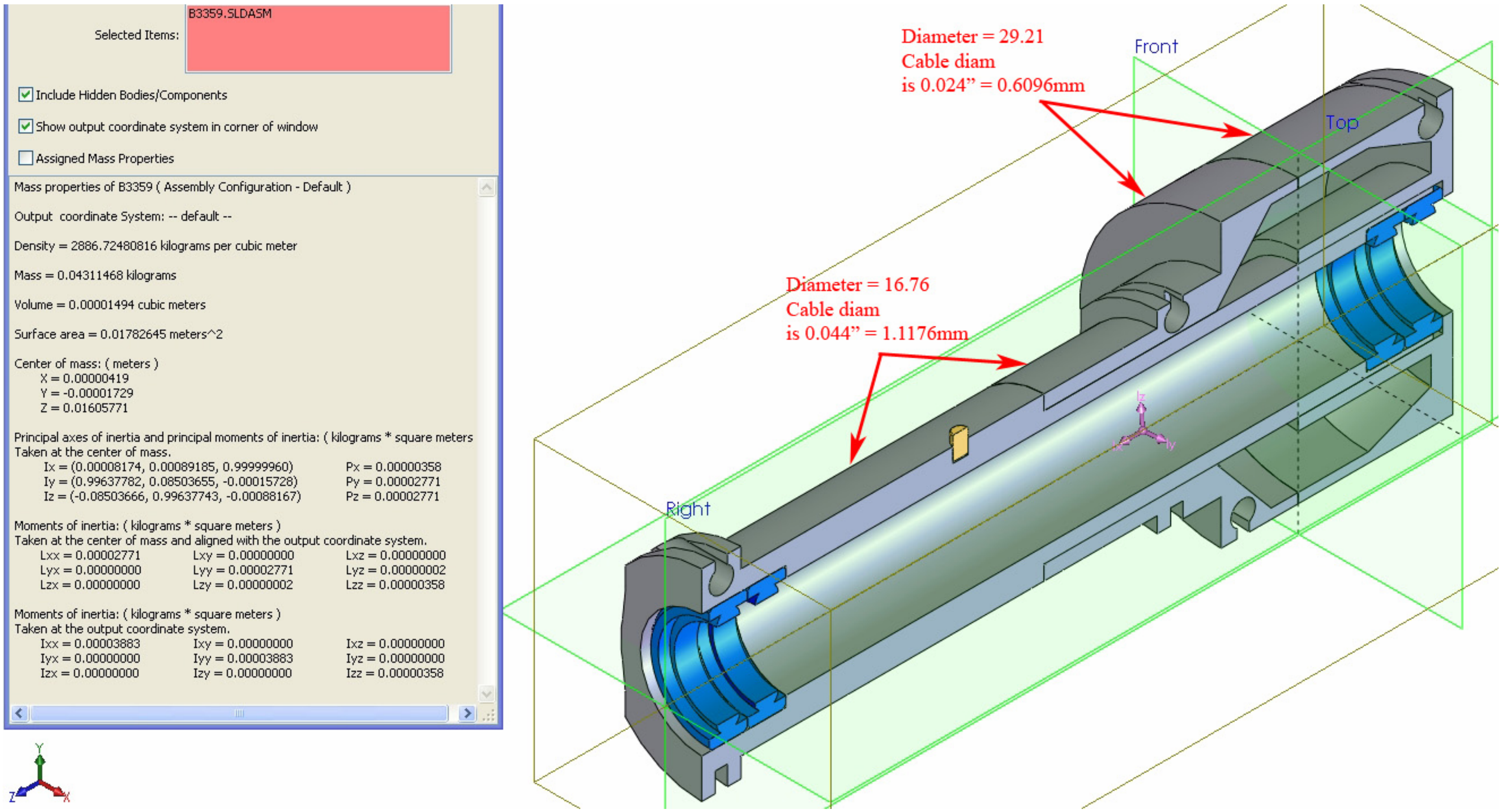

Figure 20 –Stage2-3 PulleyPinion.

Table 18 --Mass properties of B3359 (Assembly Configuration - Default)

Density = 2886.72480816 kilograms per cubic meter

Mass = 0.04311468 kilograms

Volume = 0.00001494 cubic meters

Surface area = 0.01782645 meters^2

Center of mass: (meters)

X = 0.00000419

Y = -0.00001729

Z = 0.01605771

Principal axes of inertia and principal moments of inertia: (kilograms * square meters)

Taken at the center of mass.

Ix = (0.00008174, 0.00089185, 0.99999960)	Px = 0.00000358
Iy = (0.99637782, 0.08503655, -0.00015728)	Py = 0.00002771
Iz = (-0.08503666, 0.99637743, -0.00088167)	Pz = 0.00002771

Moments of inertia: (kilograms * square meters)

Taken at the center of mass and aligned with the output coordinate system.

Lxx = 0.00002771	Lxy = 0.00000000	Lxz = 0.00000000
Lyx = 0.00000000	Lyy = 0.00002771	Lyz = 0.00000002
Lzx = 0.00000000	Lzy = 0.00000002	Lzz = 0.00000358

Moments of inertia: (kilograms * square meters)

Taken at the output coordinate system.

Ixx = 0.00003883	Ixy = 0.00000000	Ixz = 0.00000000
Iyx = 0.00000000	Iyy = 0.00003883	Iyz = 0.00000000
Izx = 0.00000000	Izy = 0.00000000	**Izz = 0.00000358**

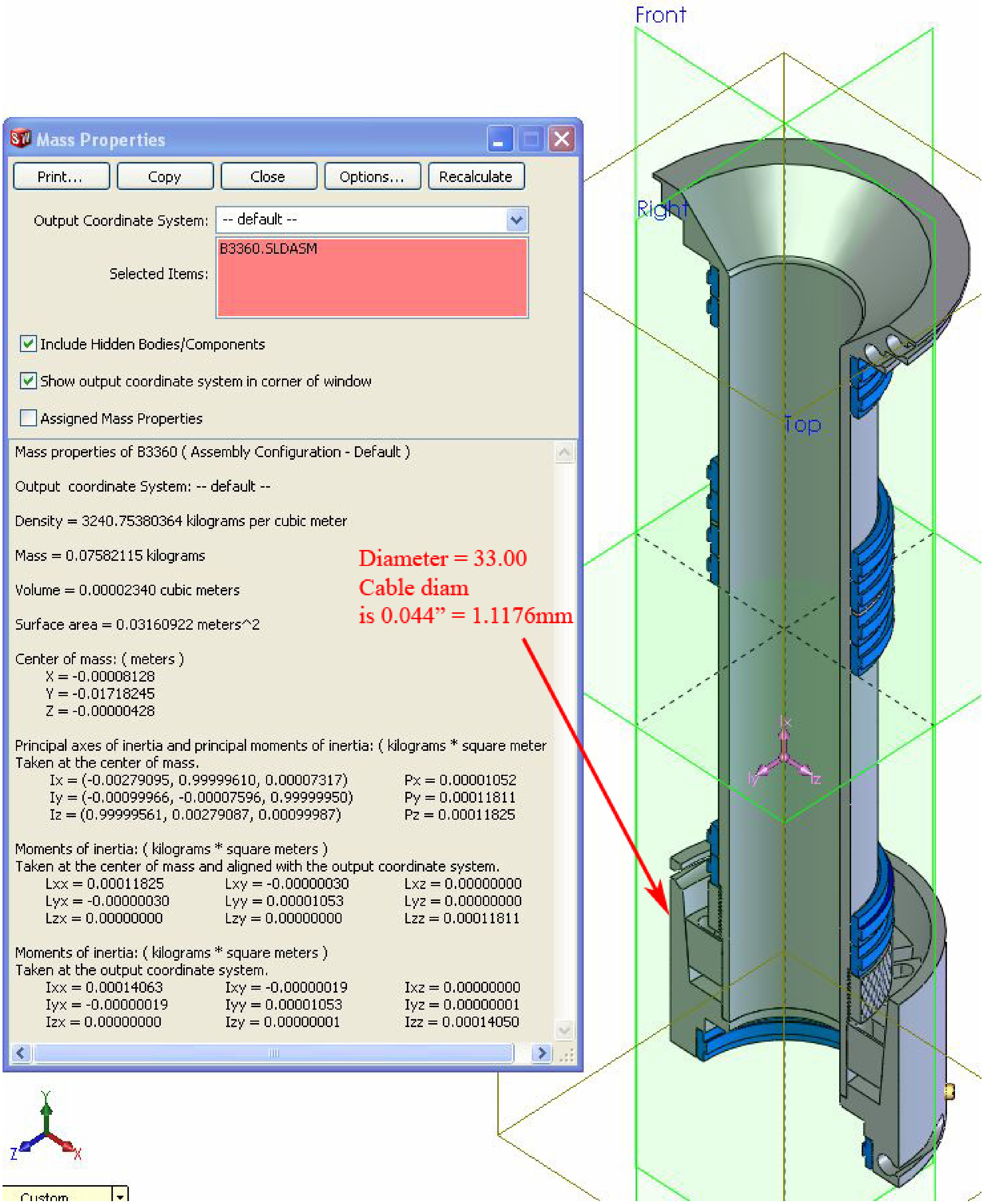

Figure 21 -- Wrist differential outer input-pulley pair drven by Motor 5.

Table 19 -- Mass properties of B3360 (Assembly Configuration - Default)

Density = 3240.75380364 kilograms per cubic meter

Mass = 0.07582115 kilograms

Volume = 0.00002340 cubic meters

Surface area = 0.03160922 meters^2

Center of mass: (meters)

X = -0.00008128

Y = -0.01718245

Z = -0.00000428

Principal axes of inertia and principal moments of inertia: (kilograms * square meters)

Taken at the center of mass.

Ix = (-0.00279095, 0.99999610, 0.00007317)	Px = 0.00001052
Iy = (-0.00099966, -0.00007596, 0.99999950)	Py = 0.00011811
Iz = (0.99999561, 0.00279087, 0.00099987)	Pz = 0.00011825

Moments of inertia: (kilograms * square meters)

Taken at the center of mass and aligned with the output coordinate system.

Lxx = 0.00011825	Lxy = -0.00000030	Lxz = 0.00000000
Lyx = -0.00000030	Lyy = 0.00001053	Lyz = 0.00000000
Lzx = 0.00000000	Lzy = 0.00000000	Lzz = 0.00011811

Moments of inertia: (kilograms * square meters)

Taken at the output coordinate system.

Ixx = 0.00014063	Ixy = -0.00000019	Ixz = 0.00000000
Iyx = -0.00000019	**Iyy = 0.00001053**	Iyz = 0.00000001
Izx = 0.00000000	Izy = 0.00000001	Izz = 0.00014050

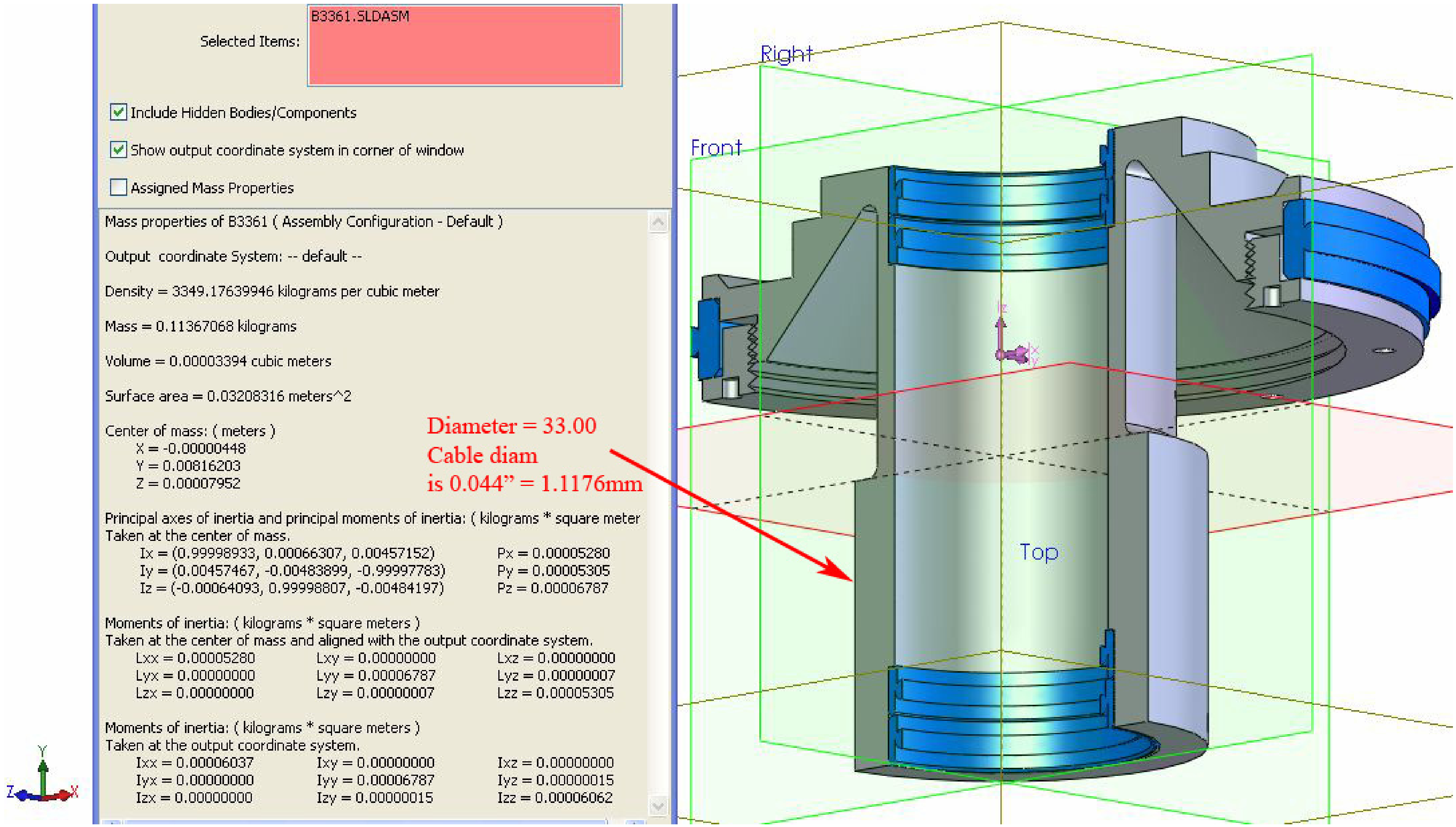

Figure 22 -- Wrist differential inner input-pulley pair driven by Motor 6.

Table 20 -- Mass properties of B3361 (Assembly Configuration - Default)

Density = 3349.17639946 kilograms per cubic meter

Mass = 0.11367068 kilograms

Volume = 0.00003394 cubic meters

Surface area = 0.03208316 meters^2

Center of mass: (meters)
X = -0.00000448
Y = 0.00816203
Z = 0.00007952

Principal axes of inertia and principal moments of inertia: (kilograms * square meters)
Taken at the center of mass.

Ix = (0.99998933, 0.00066307, 0.00457152)	Px = 0.00005280
Iy = (0.00457467, -0.00483899, -0.99997783)	Py = 0.00005305
Iz = (-0.00064093, 0.99998807, -0.00484197)	Pz = 0.00006787

Moments of inertia: (kilograms * square meters)
Taken at the center of mass and aligned with the output coordinate system.

Lxx = 0.00005280	Lxy = 0.00000000	Lxz = 0.00000000
Lyx = 0.00000000	Lyy = 0.00006787	Lyz = 0.00000007
Lzx = 0.00000000	Lzy = 0.00000007	Lzz = 0.00005305

Moments of inertia: (kilograms * square meters)
Taken at the output coordinate system.

Ixx = 0.00006037	Ixy = 0.00000000	Ixz = 0.00000000
Iyx = 0.00000000	**Iyy = 0.00006787**	Iyz = 0.00000015
Izx = 0.00000000	Izy = 0.00000015	Izz = 0.00006062

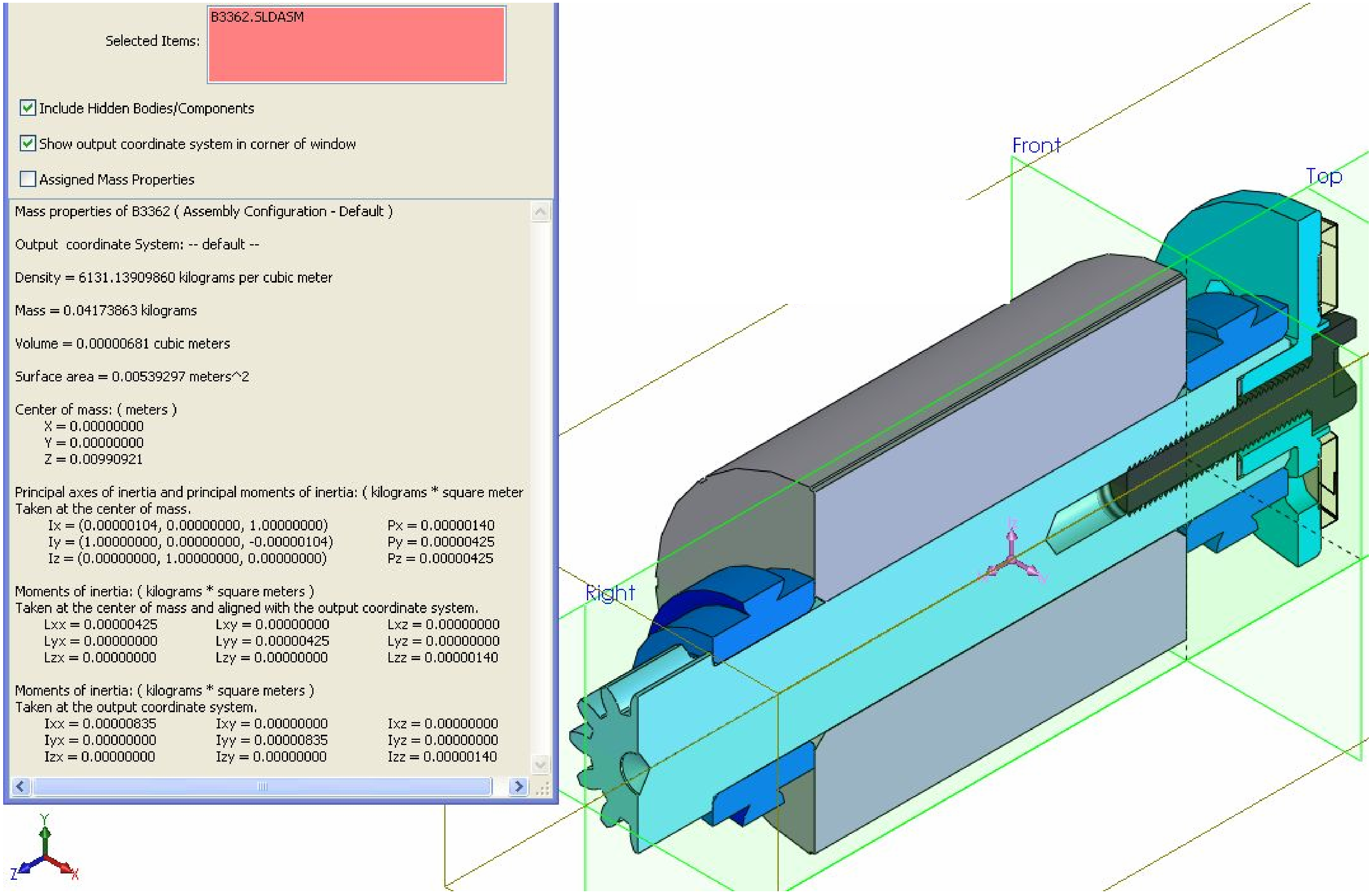

Figure 23 -- Motor-7 Inertia.

Table 21 --Mass properties of B3362 (Assembly Configuration - Default)

Density = 6131.13909860 kilograms per cubic meter

Mass = 0.04173863 kilograms

Volume = 0.00000681 cubic meters

Surface area = 0.00539297 meters^2

Center of mass: (meters)

X = 0.00000000

Y = 0.00000000

Z = 0.00990921

Principal axes of inertia and principal moments of inertia: (kilograms * square meters)

Taken at the center of mass.

Ix = (0.00000104, 0.00000000, 1.00000000)	Px = 0.00000140
Iy = (1.00000000, 0.00000000, -0.00000104)	Py = 0.00000425
Iz = (0.00000000, 1.00000000, 0.00000000)	Pz = 0.00000425

Moments of inertia: (kilograms * square meters)

Taken at the center of mass and aligned with the output coordinate system.

Lxx = 0.00000425	Lxy = 0.00000000	Lxz = 0.00000000
Lyx = 0.00000000	Lyy = 0.00000425	Lyz = 0.00000000
Lzx = 0.00000000	Lzy = 0.00000000	Lzz = 0.00000140

Moments of inertia: (kilograms * square meters)

Taken at the output coordinate system.

Ixx = 0.00000835	Ixy = 0.00000000	Ixz = 0.00000000
Iyx = 0.00000000	Iyy = 0.00000835	Iyz = 0.00000000
Izx = 0.00000000	Izy = 0.00000000	**Izz = 0.00000140**

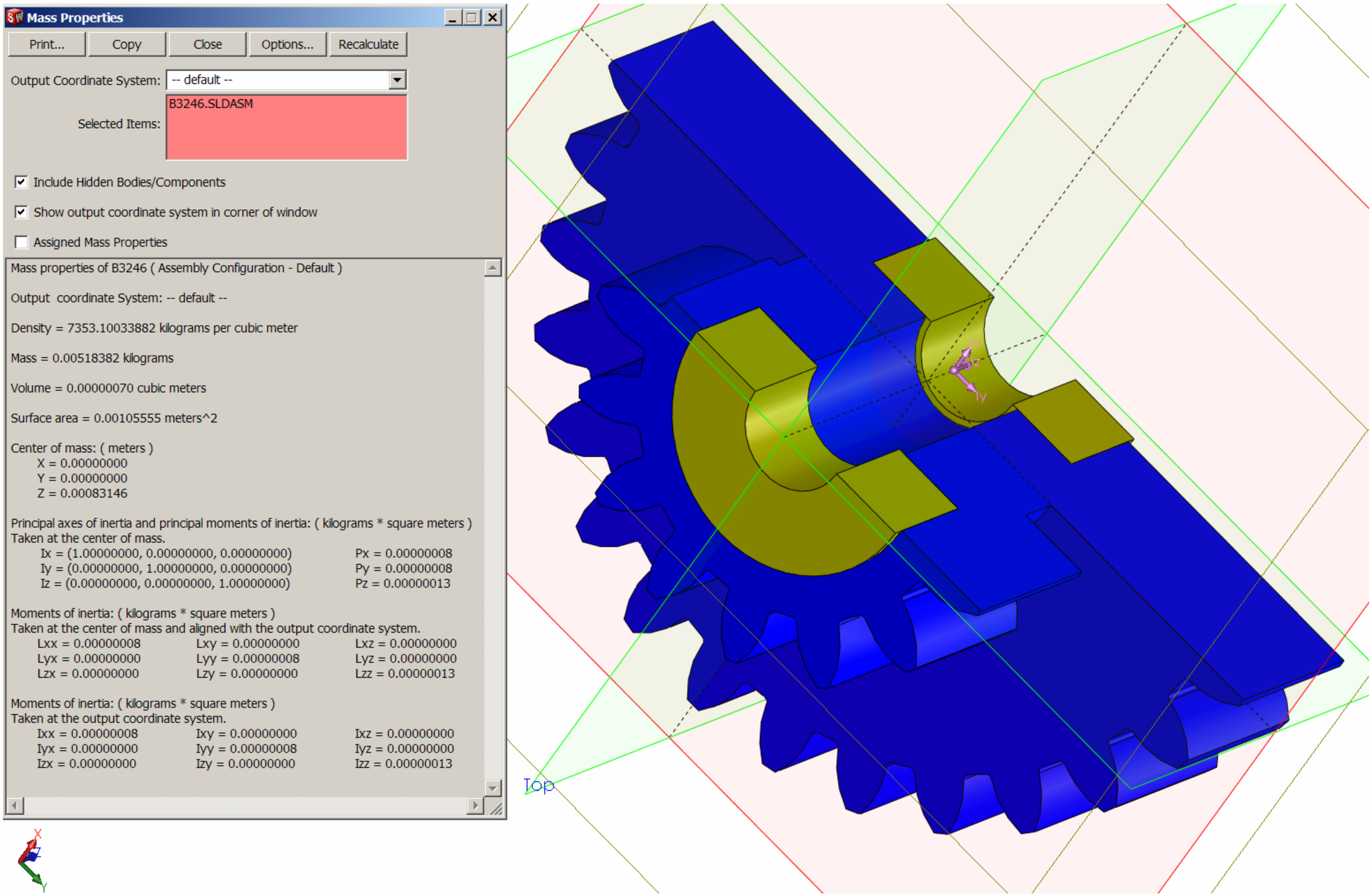

Figure 24 -- Geared 28/15 reducer.

Table 22 --Mass properties of B3246 (Assembly Configuration - Default)

Density = 7353.10033882 kilograms per cubic meter

Mass = 0.00518382 kilograms

Volume = 0.00000070 cubic meters

Surface area = 0.00105555 meters^2

Center of mass: (meters)

X = 0.00000000

Y = 0.00000000

Z = 0.00083146

Principal axes of inertia and principal moments of inertia: (kilograms * square meters)

Taken at the center of mass.

Ix = (1.00000000, 0.00000000, 0.00000000)	Px = 0.00000008
Iy = (0.00000000, 1.00000000, 0.00000000)	Py = 0.00000008
Iz = (0.00000000, 0.00000000, 1.00000000)	Pz = 0.00000013

Moments of inertia: (kilograms * square meters)

Taken at the center of mass and aligned with the output coordinate system.

Lxx = 0.00000008	Lxy = 0.00000000	Lxz = 0.00000000
Lyx = 0.00000000	Lyy = 0.00000008	Lyz = 0.00000000
Lzx = 0.00000000	Lzy = 0.00000000	Lzz = 0.00000013

Moments of inertia: (kilograms * square meters)

Taken at the output coordinate system.

Ixx = 0.00000008	Ixy = 0.00000000	Ixz = 0.00000000
Iyx = 0.00000000	Iyy = 0.00000008	Iyz = 0.00000000
Izx = 0.00000000	Izy = 0.00000000	**Izz = 0.00000013**

Table 23 -- Geometric drive data and transmission ratios.

	SAVA PN	in	CabDia	Radius Definitions	BT PN	Radii	Dia	AdjDia	Radius	Ratios		
			mm				mm	mm	mm			
M1	SN2047	0.044	1.1176	Pinion of Motor Rotor	B2072	r_m	16.88	18.00	9.00	N_s1	3.6670	
	SN2047	0.044	1.1176	Pulley of Grenade	B2198	R_pp	64.88	66.00	33.00			
	SN2054	0.054	1.3716	Pinion of Grenade	B2198	r_pp	20.63	22.00	11.00	N_s2	11.4538	
	SN2054	0.054	1.3716	BasePulley	B1898	R_base	250.63	252.00	126.00			
										N = N_s1 * N_s2	**42.0013**	**M1**
M2&3	SN2047	0.044	1.1176	Pinion of Motor Rotor	B2072	r_m	16.88	18.00	9.00	N_s1	3.6670	
	SN2047	0.044	1.1176	Pulley of Grenade	B2198	R_pp	64.88	66.00	33.00			
	SN2054	0.054	1.3716	Pinion of Grenade	B2198	r_pp	20.63	22.00	11.00	N_s2	7.7041	
	SN2054	0.054	1.3716	Pulley of Differential Inputs	B1910	R_diff	168.13	169.50	84.75			
										N = N_s1 * N_s2	**28.2510**	**M1 & M2**
M4	SN2047	0.044	1.1176	Pinion of Motor Rotor	B2072	r_m	16.88	18.00	9.00	N_s1	3.4448	
	SN2047	0.044	1.1176	Pulley of ElbowPulleyPinion	B2196	R_pp	60.88	62.00	31.00			
	SN2054	0.054	1.3716	Pinion of ElbowPulleyPinion	B2196	r_pp	18.53	19.90	9.95	N_s2	5.2258	
	SN2054	0.054	1.3716	ElbowOutputHalfPulleys	B2186/7	R_diff	102.63	104.00	52.00			
										N = N_s1 * N_s2	**18.0016**	**M4**
M5&6	SN2019	0.018	0.4572	Pinion of Motor Rotor	B3357	r_m	12.70	13.16	6.58	N_s1	2.6599	
	SN2019	0.018	0.4572	Pulley of Stage1-2 PP	B3358	R_pp12	34.54	35.00	17.50			
	SN2024	0.024	0.6096	Pinion of Stage1-2 PP	B3358	r_pp12	15.00	15.61	7.80	N_s2	1.9103	
	SN2024	0.024	0.6096	Pulley of Stage2-3 PP	B3359	R_pp23	29.21	29.82	14.91			
	SN2047	0.044	1.1176	Pinion of Stage2-3 PP	B3359	r_pp23	16.76	17.88	8.94	N_s3	1.9084	
	SN2047	0.044	1.1176	Pulley of Differential Inputs	B3360/1	R_diff	33.00	34.12	17.06			
										N = N_s1 * N_s2 * N_s3	**9.6973**	**M5&6**
M7	N/A			Pinion of Motor Rotor	B3362	r_m	12			N_s1	2.3333	
	N/A			Idler Gear	B3246	R_pp	28					
	N/A			Pinion of Idler Gear	B3246	r_pp	15			N_s2	6.4000	
	N/A			Output Gear	B3347	R_out	96					
											14.9333	**M7**

Table 24 – Inertia of discrete drive components translated to the equivalent inertias (reflected) at each rotor.

		BTech	part inertia	cbl masses	cbl radius	inertia w/cbls	stg1	stg2	stg3	inertia@rotor
		PN	kg-m^2	kg	m	kg-m^2				kg-m^2
m1234	**rotor**	B3363	0.00010064	0.011	0.009	0.00010242	1.00	1.00	1.00	0.00010242
m123	**grenade-pp X2**	B2237	0.00009183	0.013	0.011	0.00018681	3.67	1.00	1.00	0.00001389
m2/3	**diff input**	B2331	0.00338288	0.013	0.075	0.00352913	3.67	11.45	1.00	0.00000200
m4	**elbow pp**	B2284	0.00005105	0.008	0.010	0.00005263	3.44	1.00	1.00	0.00000444
m56	**rotor**	B3357	0.00001565	0.001	0.007	0.00001574	1.00	1.00	1.00	0.00001574
m56a	**S12-pp**	B3358	0.00000558	0.002	0.008	0.00000582	2.66	1.00	1.00	0.00000082
m56b	**S23-pp**	B3359	0.00000358	0.003	0.009	0.00000406	2.66	1.91	1.00	0.00000016
m5	**diff input (outer)**	B3360	0.00001053	0.003	0.018	0.00001247	2.66	1.91	1.91	0.00000013
m6	**diff input (inner)**	B3361	0.00006787	0.003	0.013	0.00006888	2.66	1.91	1.91	0.00000073
m7	**rotor**	B3362	0.00000140	0.000	0	0.00000140	1.00	1.00	1.00	0.00000140
m7i	**rotor**	B3246	0.00000013	0.000	0	0.00000013	2.33	1	1.00	0.00000002

Table 25 – Combined total drive inertias by motor number reported both at the motor rotor and at the drive output.

Drive Number	Summed Components from Previous Table	Inertia@Rotor	Drive Ratio	Inertia@Output
		kg-m^2		kg-m^2
M1	**m1234 + m123**	0.00011631	42.00	0.20518962
M2	**m1234 + m123 + m2/3**	0.00011831	28.25	0.09442836
M3	**m1234 + m123 + m2/3**	0.00011831	28.25	0.09442836
M4	**m1234 + m4**	0.00010686	18.00	0.03462804
M5	**m56 + m56a + m56b + m5**	0.00001685	9.70	0.00158448
M6	**m56 + m56a + m56b + m6**	0.00001745	9.70	0.00164089
M7	**m7+m7i**	0.00000142	14.93	0.00031753