Calculating Motorcycle Center of Mass

1 Introduction

The procedure below shows how to determine a motorcycle *center of mass* (or "center of gravity") by weighing both tire contact forces on a horizontal surface, then on a slope. Although the method will work on any two-wheeled vehicle, I applied it to my 2013 BMW R1200GS. The analysis is simple statics: force and moment balances. But the geometry was a little tricker than I expected.

2 Horizontal Weighing

2.1 Horizontal Free Body Diagram

The free-body diagram of a two-wheeled vehicle resting on a horizontal surface is shown in Figure 1.

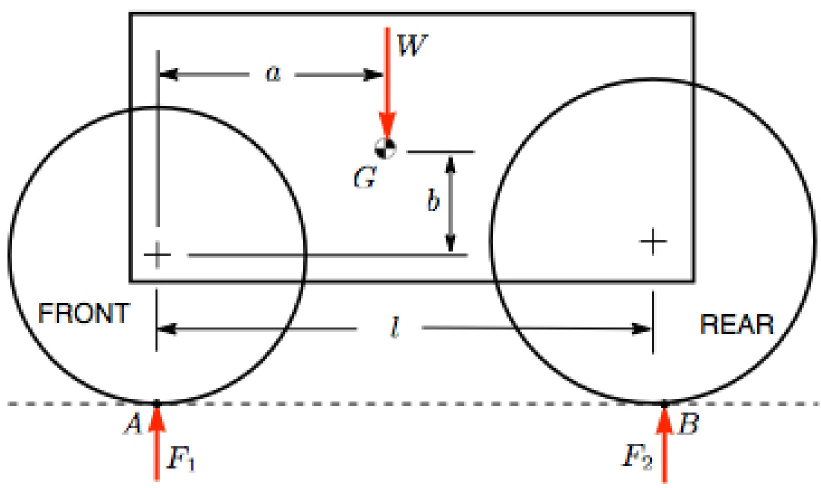

Figure 1: Free body diagram of motorcycle resting on a level surface.

The notation in Figure 1 is:

$$
\begin{aligned}
F_1 &= \text{weight at front tire} \\
F_2 &= \text{weight at rear tire} \\
W &= \text{total weight of motorcycle} \\
A &= \text{front tire contact point} \\
B &= \text{rear tire contact point} \\
G &= \text{center of mass of motorcycle} \\
l &= \text{wheelbase of motorcycle} \\
a &= \text{horizontal distance from front axle to center of mass} \\
b &= \text{vertical distance from front axle to center of mass}
\end{aligned}
$$

Note that the front and rear tires can be of different radii, which is usually true. Next is the analysis stemming from Figure 1.

2.2 Horizontal Analysis

The procedure is to first perform a force balance in the vertical direction, thus

$$\Sigma F_v \implies F_1 + F_2 - W = 0, \tag{1}$$

then a moment balance around point A:

$$\Sigma M_A \implies F_2\, l - W a = 0. \tag{2}$$

Forces F_1 and F_2 can easily be determined with a "bathroom" scale. Equation (1) can be rearranged to yield

$$\boxed{W = F_1 + F_2} \tag{3}$$

Although this is hardly a momentous result, once W is known, equation (2) can yield distance a:

$$\boxed{a = \frac{F_2\, l}{W}} \tag{4}$$

So now we know the horizontal distance from the front axle to the center of mass G. Now for the vertical distance b from the front axle to the center of mass. This requires weighing of the rear tire contact force (don't need the front) on an "angle."

3 Angled Analysis

Perform a similar procedure to Section 2.2, but with the motorcycle on a slope.

3.1 Angled Free Body Diagram

The free-body diagram of a two-wheeled vehicle resting on a horizontal surface is shown in Figure 2.

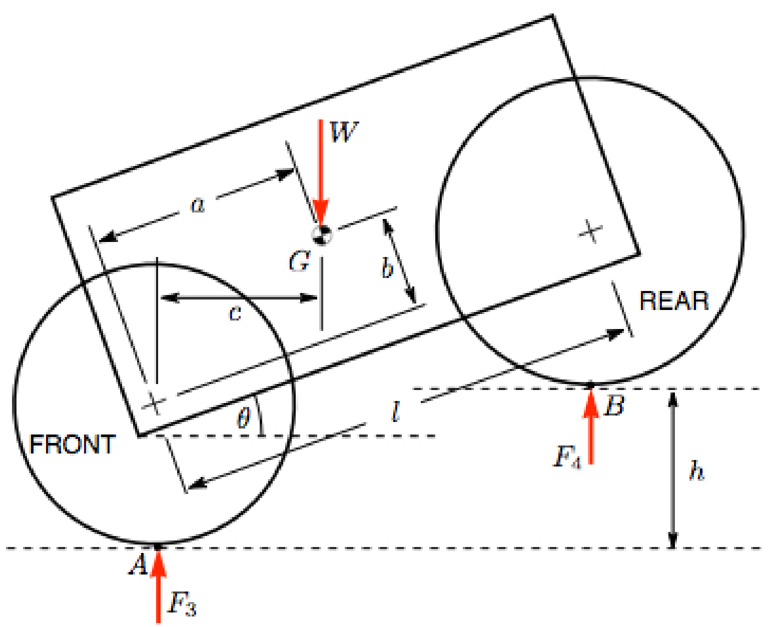

Figure 2: Free body diagram of motorcycle resting on an angled surface.

The quantities in Figure 2 are the same as before, however forces F_3 and F_4 will be different—this is the point of weighing on an angle. Also distance c has been added—this will simplify things a bit. The height of the rear wheel is h, and the angle of the surface is θ.

3.2 Angled Analysis

Perform similar force and moment balances as before—weight W is now known, and tire force F_4 must be measured with a scale (don't need F_3). Also angle θ must be known, although it can be computed from the measured height of the rear wheel.

$$\Sigma F_v \implies F_3 + F_4 - W = 0, \tag{5}$$

Since W is known, angled front tire force F_3 could be found from equation (5), but we really aren't interested in that, so equation (5) is not really useful. Just a habit...

The moment balance around point A is far more useful,

$$\Sigma M_A \implies F_4\, l \cos\theta - W c = 0 \tag{6}$$

We need to express distance c in terms of a and b. The relationship is:

$$c = a\cos\theta - b\sin\theta \tag{7}$$

Substituting (7) into (6) ultimately yields

$$\boxed{b = \frac{(W a - F_4\, l)\cos\theta}{W \sin\theta}}, \tag{8}$$

which, since a is known from result (4), is a usable result.

Finally, angle θ can be determined from a measurement of rear wheel height h and known wheelbase l,

$$\sin\theta = \frac{h}{l} \quad\implies\quad \boxed{\theta = \sin^{-1}\frac{h}{l}} \tag{9}$$

This center of mass result will be more accurate the greater the angle θ.

4 An Example: the 2013 BMW R1200GS

My '13 GSW has SWMotech upper and lower crash bars, Touratech sidecase and topcase racks, so I suppose it's a bit heavier than showroom stock. Also, the example below was with about 60 miles on a full tank of fuel, so probably around one gallon less than full.

I actually wrote a MATLAB (scientific computing package) script to do this simple computation, and here's what I got:

$F_1 = 300$ lb, weight at front tire (horizontal)
$F_2 = 289$ lb, weight at rear tire (horizontal)
$W = 589$ lb, heavier than stock due to racks, *etc.*
$h = 20.5$ in, the highest I raised up the rear of the bike
$F_4 = 247$ lb, angled rear tire force (less than horizontal due to angle)
$\theta = 20.2^\circ$, angle of bike with rear wheel raised

Now for the real result of interest, the location of the center of mass relative to the front axle:

$a = 29.1$ in, center of mass is this far *behind* the front axle
$b = 11.5$ in, center of mass is this far *above* the front axle

I've tried to show the location of the center of mass G on the photo of the bike in the post. Anyway, kind of interesting...