

Chapter 2 HW I Hints and Answers

Problem 1. This problem is very similar to Example 2.5, which was done in class. I defined fixed frame XYZ as frame A , and rotated frame xyz as frame B . Note that our convention is that a “CW” rotation is negative, and a “CCW” rotation is positive.

The rotation is a 2-1 rotation with $\theta_1 = -30^\circ$ and $\theta_2 = 45^\circ$. The combination $[R] = [R_1][R_2]$ matrix relates vectors between in frames B and A .

a. The problem is: given ${}_A\{r\}_i$, find ${}_A\{r\}_f$. Note that vector $\{r\}_i$ has the same relationship to frame A as vector $\{r\}_f$ does to frame B . Using matrix $[R]$ my results were

$${}_A\{r\}_f = \begin{bmatrix} 0.3536 \\ 2.5981 \\ -2.4749 \end{bmatrix} = 0.3536\mathbf{a}_1 + 2.5981\mathbf{a}_2 - 2.4749\mathbf{a}_3 \quad (1)$$

b. Here the problem is given ${}_B\{p\}$ find ${}_A\{p\}$. Again using rotation matrix $[R]$:

$${}_A\{p\} = \begin{bmatrix} 0.3282 \\ 2 \\ 4.5708 \end{bmatrix} \quad (2)$$

c. Use $[R]$ and its inverse to transform $\mathbf{j}$ and $\mathbf{K}$ into the B and A frames. My results are:

$$\mathbf{v} = 1.4142\mathbf{a}_1 - 3.4641\mathbf{a}_2 + 2.6142\mathbf{a}_3 = -0.8485\mathbf{b}_1 - 4.4243\mathbf{b}_2 + 0.7348\mathbf{b}_3 \quad (3)$$

where for (a), (b), and (c): $\mathbf{a}_1\mathbf{a}_2\mathbf{a}_3$ are **IJK** and $\mathbf{b}_1\mathbf{b}_2\mathbf{b}_3$ are **ijk**.

Problem 4. For this problem just find velocity, not acceleration. The velocity can be obtained by taking the derivative of $\{r\}$ with respect to t using $\{q\}$ and the chain rule (note that vector $\{q\} = [\theta_1 \ \theta_2]^T$);

$$\{v\} = \{\dot{r}\} = \underbrace{[r_q]}_{\text{Jacobian}} \{\dot{q}\} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} L_1c_1 & L_2c_2 \\ L_1s_1 & L_2s_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \quad (4)$$

Problem 9. Again let fixed frame $XYZ = A$, and rotated frame $xyz = B$. The initial coordinates of point C are $({}_A\mathbf{r}_c)_i = 3\mathbf{a}_1 + 2\mathbf{a}_2 + \mathbf{a}_3$. Frame B is defined by a 1-2 rotation, where $\theta_1 = -15^\circ$ and $\theta_2 = 45^\circ$. Use the same technique as in Problem 1:

$${}_A\{r_c\}_f = [R]^T {}_A\{r_c\}_i = \begin{bmatrix} 2.8284 \\ 1.5658 \\ -1.8837 \end{bmatrix} \quad (5)$$

Problem 11. With the fixed frame XYZ denoted by F , the axis about which the box is rotated can be considered to be the z axis of rotated frame $xyz = G$, which is created by a 2-rotation by 50.19° followed by a 1-rotation by -14.36° , then line OB is along the “3” axis of frame G . The rotation of the box is then a 3-rotation by 30° , so following the approach of the previous problems we find

$${}_F\{r_A\}_f = [R_2]^T [R_1]^T [R_3]^T [R_1][R_2]{}_F\{r_A\}_i = \begin{bmatrix} 0.1792 & -0.0044 & 2.6866 \end{bmatrix}^T \quad (6)$$

Problem 14. Consider frame A fixed to ground, while frame B is fixed to the aircraft. Both frames are coincident at this instant (as shown in Figure 2). We will also *express* the angular acceleration in fixed frame A (we could express it in any frame).

The angular velocity of the propellor (P) relative to ground (A) can be expressed

$${}^A\boldsymbol{\omega}^P = {}^A\boldsymbol{\omega}^B + {}^B\boldsymbol{\omega}^P \quad (7)$$

The angular velocity of the propellor is due to three separate rotations: (1) the rotation ω_1 of the airplane around its curved trajectory (its speed is 880 ft/s), (2) the upward pitching motion ω_2 of the airplane, and (3) the spinning ω_3 of the propellor.

The angular acceleration ${}^A\boldsymbol{\alpha}^P$ is the time derivative of ${}^A\boldsymbol{\omega}^P$ relative to frame A . Since all the ω_i and $\mathbf{a}_3$ are constant, the only time derivatives will come from differentiating the $\mathbf{b}_1$ and $\mathbf{b}_2$ unit vectors of (23) with respect to frame A . These unit vectors can be differentiated using the *Transport Theorem*. The final result is:

$${}^A\boldsymbol{\alpha}^P = \omega_1\omega_2\mathbf{a}_1 - \omega_1\omega_3\mathbf{a}_2 - \omega_2\omega_3\mathbf{a}_3 \quad (8)$$

$$= 0.0293\mathbf{a}_1 - 122.9\mathbf{a}_2 - 41.9\mathbf{a}_3 \frac{\text{rad}}{\text{sec}^2} \quad (9)$$

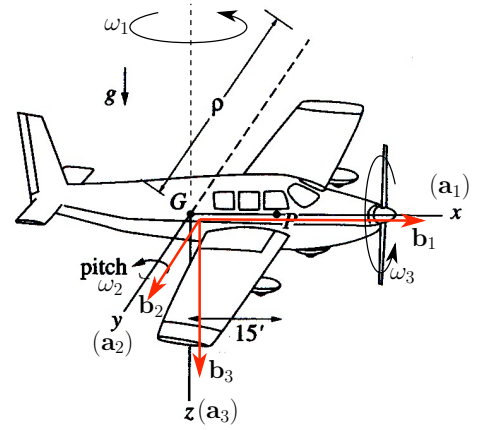


Figure 1: Airplane of Problem 14.

Problem 20. Again, the *Transport Theorem* is the key to this problem. The angular velocity of the flywheel can be expressed as the sum of relative angular velocities like Problem 14, then the differentiation relative to ground of each term can be done with the Transport Theorem.

The frame assignments are shown in Figures 3a and 3b on the next page. Frame A is fixed to ground. Frame B is fixed to the outer gimbal, while frame C is fixed to the inner gimbal. Letter F will designate the flywheel (no need to fix a frame to the flywheel). Frames A and B are coincident at this instant.

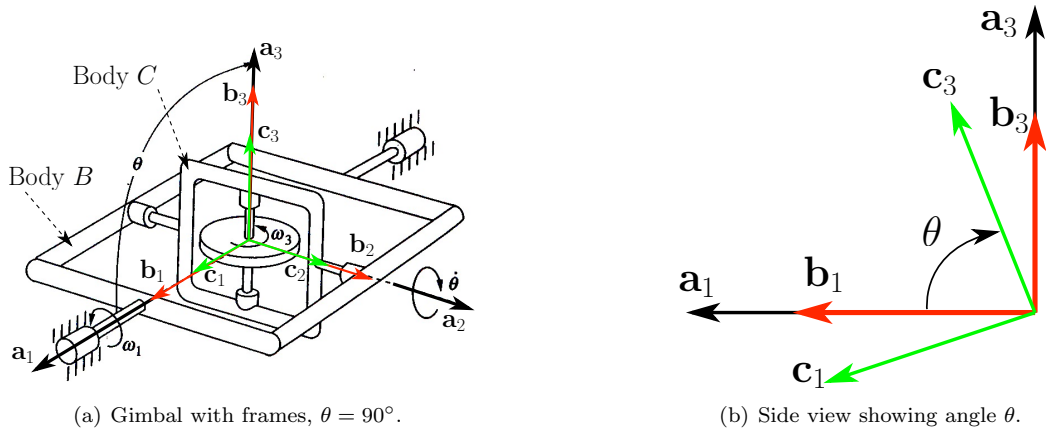


Figure 2: Frame assignments for Problem 20.

The angular velocity of the flywheel is given by

$${}^A\boldsymbol{\omega}^F = {}^A\boldsymbol{\omega}^B + {}^B\boldsymbol{\omega}^C + {}^C\boldsymbol{\omega}^F = \omega_1\mathbf{a}_1 - \omega_2\mathbf{b}_2 + \omega_3\mathbf{c}_3 \quad (10)$$

where $\omega_2 = \dot{\theta}$ and the unit vectors are related by using Figure 3b.

The final result is

$${}^A_A\boldsymbol{\alpha}^F = -1.8\mathbf{a}_1 - 3\mathbf{a}_2 - 1517\mathbf{a}_2 \quad (11)$$

$$\boxed{= -1.8\mathbf{a}_1 - 1520\mathbf{a}_2 \quad \frac{\text{rad}}{\text{sec}^2}} \quad (12)$$