Report 3 - Time-Dependent Flows

The flow past a circular cylinder can remain steady and symmetric at modest Reynolds number, Re, or it can exhibit a time-periodic behavior if the Reynolds number is sufficiently high. Experimentally it has been observed that the flow past a circular cylinder remains symmetric and steady up to $30 \leq Re_{uns} \leq 40$, where Re_{uns} is the Reynolds number where unsteady flow is first observed. Above this Reynolds number, the flow becomes unsteady and exhibits a vortex shedding process that produces a Karman vortex street downstream of the cylinder.

Empirically, it has been observed that there is a correlation between the Reynolds number and the vortex shedding frequency. In a non-dimensional form, the shedding frequency is simply the Strouhal number

$$St = \frac{fD}{U_\infty}, \tag{1}$$

where f is the shedding frequency, D is the cylinder diameter and U_∞ is the free-stream (scaling) velocity.

It has been suggested that the empirical relationship between the Strouhal and Reynolds numbers is

$$St = 0.1959 - \frac{3.811}{Re} \quad for \;\; Re_{uns} \leq Re \leq 130, \tag{2}$$

and

$$St = 0.2064 - \frac{5.173}{Re} \quad for \;\; 130 \leq Re \leq 250. \tag{3}$$

In order to verify these correlations, complete the FE2D time-dependent Navier Stokes solver, and perform the computational studies outlined below. Report your findings using the report format from the first programming exercise. Note, be sure to include the governing equations and concomitant finite element weak form in your formulation section.

The non-dimensional conservation equations for isothermal, time-dependent, laminar, incompressible, viscous flow are

$$\nabla \cdot \mathbf{u} = 0, \tag{4}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{Re}\nabla^2 \mathbf{u} + \mathbf{f}, \tag{5}$$

where $\mathbf{u} = (u, v)$ is the velocity, Re is the Reynolds number, $\mathbf{f}$ is the body force, and p is the pressure.

The flow configuration for the computational studies is shown below in Figure 1. The initial conditions are

$$\begin{aligned} u(\mathbf{x}, \mathbf{0}) &= 1, \\ v(\mathbf{x}, \mathbf{0}) &= 0 \end{aligned} \tag{6}$$

For all the computational experiments, use the "tow-tank" conditions as shown in Table 1. Perform simulations for $Re = 25,\ 50,\ 100,\ 150$ using a consistent mass matrix, and the second-order semi-implicit method ($\theta_k = 0.5$ in the FE2D control). Perform one reference calculation at $Re = 100$ using backward-Euler ($\theta_k = 1.0$) and a lumped mass matrix. Compare the results to the second-order semi-implicit results at this Reynolds number.

Use time-history points to record the velocity as a function of time in the cylinder wake at $x = 1D,\ 2D,\ 3D,\ 4D$ along the centerline of the grid. For each Reynolds number, report the Strouhal number vs. Reynolds number and compare the results to the Strouhal number based on the correlations above – this is most easily done with a plot of Strouhal number vs. Reynolds number.

For this non-dimensional flow problem, the Strouhal number is simply $1/T$ where T is the period of oscillation for the y-velocity component in the cylinder wake. The period of oscillation may be found by finding the zero-crossing for the y-velocity at each of the time-history points during steady-periodic vortex shedding for a duration of *at least* 1-period. For each calculation, provide plots of

1. The kinetic energy vs. time.

2. The RMS divergence vs. time.

3. The x- and y-velocity at the time-history points above.

4. Optionally, provide snapshots of vorticity, stream-function and the nodal pressure.

Note that the RMS divergence and kinetic energy are written to the file called "glob" by default from FE2D. You may plot this file directly using gnuplot or other similar tool.

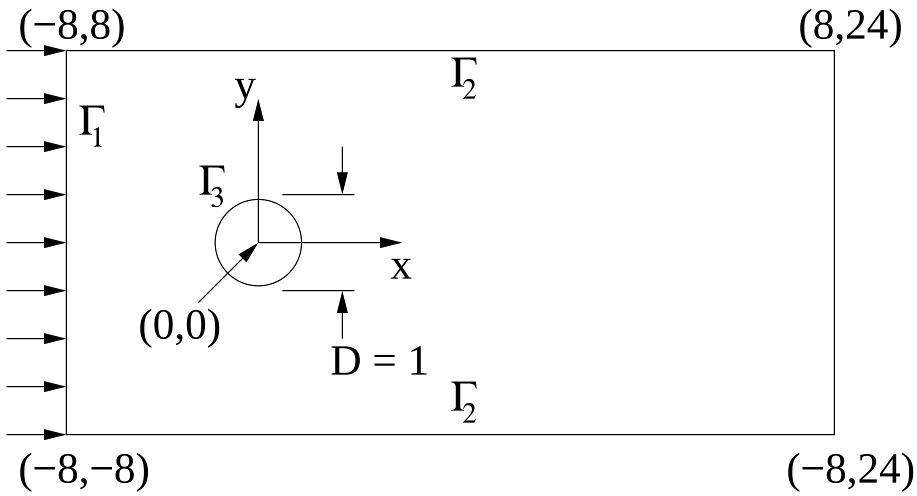

Figure 1: Tow-tank flow configuration (not to scale).

Boundary	X-Velocity (u)	Y-Velocity (v)
Γ_1	$u = 1$	$v = 0$
Γ_2	$u = 1$	$v = 0$
Γ_3	$u = 0$	$v = 0$

Table 1: Tow-tank boundary conditions.