Report 1 - Time-Dependent Heat Conduction

Complete the FE2D time-dependent conduction solver, and perform the computational studies outlined in parts A and B below. Report your findings using the report format outlined below.

a) Verification

Perform a verification study using the conduction solver in FE2D to solve the one-dimensional conduction problem shown in Figure 1. The governing equation for this problem is

$$\frac{\partial T}{\partial t} = \kappa \left\{ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right\}, \tag{1}$$

where the thermal diffusivity is $\kappa = 8.418 \times 10^{-5} \quad m^2/s$, and $\kappa = k/\rho C_p$. The initial temperature is constant at $T(x, 0) = 100\ ^oC$.

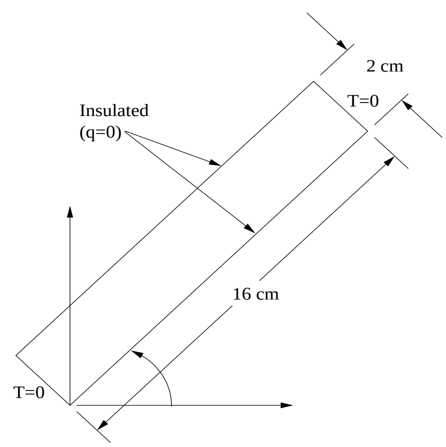

Figure 1: Verification problem (not to scale).

In order to verify that the conduction solver is functioning properly, use the exact solution for the problem. Check the numerical solution using the lumped, consistent and "high-order" mass matrices with a variety of time-integration options (i.e., $\theta = 0,\ 1/2,\ 1$) against the exact solution at $t = 1,\ 5,\ 10s$.

From your numerical experiments, is there any difference between the time-integration methods or mass-matrix formulations? How sensitive is the solution to time step and mesh size? Summarize the results of the verification study in the report format outlined below.

> **Hint:** The exact solution for this problem may be developed using separation of variables for the problem written in terms of a local coordinate system attached to the bar, i.e., a one-dimensional heat-conduction problem.

b) The Fin

A long steel container is used to enclose a region where a high temperature chemical reaction occurs. To keep the temperature of the container from overheating (and melting), aluminum fins have been placed along the outside of the container. Figure 2 shows a cross section of the container wall and fin.

At the start of the reaction process, the container and fin are at a uniform temperature of 39 ^oC. The reaction results in an effective heat flux of 3000 W/m^2 on the interior of the container. The convective heat transfer coefficient on the exterior of the container is 16 $W/m^2 \cdot^o C$. The material properties are listed in the table below.

The governing equation for this problem is

$$\rho C_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[k_{xx}\frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[k_{yy}\frac{\partial T}{\partial y}\right] \quad (2)$$

where ρ is the mass density, C_p is the specific heat, and (k_{xx}, k_{yy}) are the thermal conductivities.

Will the container ultimately melt, or are the cooling fins sufficient to prevent this? Again, present your analysis following the report format below.

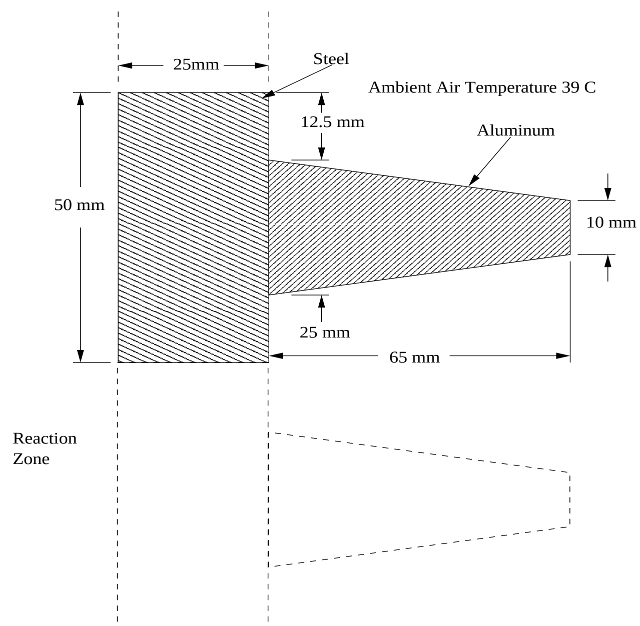

Figure 2: The cooling fin (not to scale).

Parameters/Material Properties	Steel	Aluminum
Mass density (kg/m^3)	7833.0	2760.0
Specific Heat ($J/kg \cdot^o C$)	486.0	1050.0
Thermal conductivity ($W/m \cdot^o C$)	41.0	183.0

Report Format

This format should be followed for all reports.

1. Title page with your name and date.

2. Problem Statement: A statement of the problem that defines the specific problem under consideration. The problem definition includes the governing equations, boundary conditions, initial conditions, and all simplifying assumptions.

3. Formulation: A description of the finite element formulation along with a brief outline of the software used to solve the problem, and a definition of all the input-output variables follows the problem statement. The program description should present the type of finite element used and highlight any dependencies on linear algebra libraries, etc.

4. Results and Conclusions. This section should include a description of the finite element meshes, boundary conditions, and all modeling assumptions made to simulate the problem. All pertinent results including any graphics/plots should be presented here. The comparison of results with other solutions should be presented and discussed here. Where appropriate, discuss the results and present summary conclusions.

5. References.

6. Appendix A. All calculations necessary for the analysis should be reported here, e.g., hand calculations, exact solutions, etc.

7. Appendix B:

   - Listing of code fragments that you have developed.
   - Attach a listing of the "data echo" that is printed to the screen by FE2D.
   - Attach any relevant output data.