ME561-1

Given a domain Ω with boundary Γ, develop the weak form for

$$-\nabla \cdot (k \nabla T) = Q''' \ in \ \Omega$$

with boundary conditions

$$T = \hat{T} \ on \ \Gamma_T,$$

and heat flux conditions

$$-(k \nabla T) \cdot \mathbf{n} = \hat{q}_n \ on \ \Gamma_q,$$

and $\Gamma = \Gamma_T \bigcup \Gamma_q$.

Discretize the problem using the 4-node bilinear quadrilateral element and develop the element-level operators (symbolically) in terms of the element shape functions.

Partial solution:

$$\int_\Omega \nabla w \cdot (k \nabla T) = \int_\Omega w Q''' - \oint_\Gamma w (k \nabla T) \cdot \mathbf{n}$$

ME561-2

Given a domain Ω with boundary Γ, develop the weak form for

$$-\nabla \cdot (k\nabla T) = Q''' \; in \; \Omega$$

with boundary conditions

$$T = \hat{T} \; on \; \Gamma_T,$$

and heat flux conditions

$$-(k\nabla T) \cdot \mathbf{n} = h(T - T_\infty) \; on \; \Gamma_q,$$

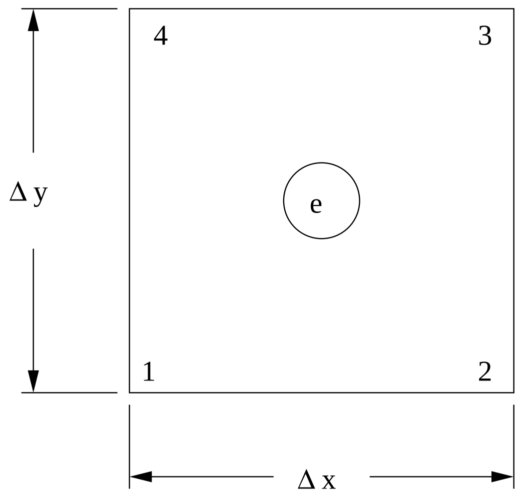
and $\Gamma = \Gamma_T \bigcup \Gamma_q$.

Show that the global equations have the form $[K]\{T\} + [H]\{T\} = \{Q\}$.

Discretize the problem using the 4-node bilinear quadrilateral element and develop all the element-level operators (symbolically) in terms of the shape functions.

ME561-3

Using the results from problem ME561-2, develop all the element-level operators in terms of a generic quadrilateral element with edge lengths Δx and Δy. Present the element level operators, K^e_{IJ}, H^e_{IJ}, and Q^e_I in terms of Δx and Δy.

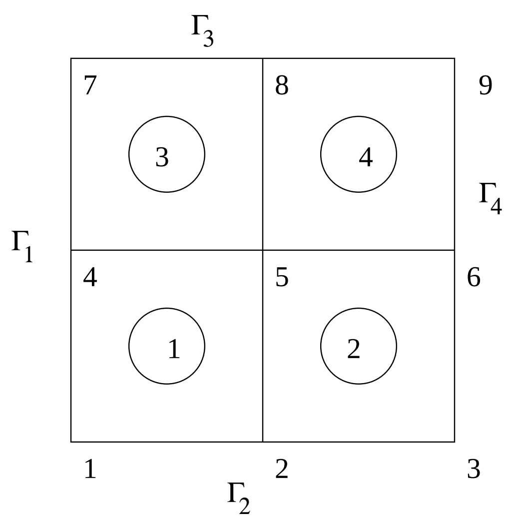


ME561-4

Consider steady-state heat conduction in a square domain discretized as shown by the mesh below. Assume that the medium has a thermal conductivity k and a uniform heat generation rate Q_o'''. The grid has uniform mesh spacing, i.e., $\Delta x = \Delta y = a$.

The boundary conditions are as follows:

Surface	Boundary Conditions	
Γ_1	Input heat flux	$-(k\nabla T)\cdot\mathbf{n} = q_o$
Γ_2	Insulated	$-(k\nabla T)\cdot\mathbf{n} = 0$
Γ_3	Convective flux	$-(k\nabla T)\cdot\mathbf{n} = h(T - T_\infty)$
Γ_4	Fixed Temperature	$T = \hat{T}_4$



Develop the element-level operators in terms of the mesh spacing, a, for the conduction problem. Assemble the complete finite element equations at nodes $1, 5, 7, 9$ – present the equations in terms of the specified heat flux, convective coefficient, thermal conductivity and heat generation rate.

ME561-5

Given a domain Ω with boundary Γ, develop the weak form for

$$\rho C \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k_{xx}\frac{\partial T}{\partial x} + k_{xy}\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial y}\left(k_{yx}\frac{\partial T}{\partial x} + k_{yy}\frac{\partial T}{\partial y}\right) + Q'''$$

with boundary conditions

$$T = \hat{T} \; on \; \Gamma_T,$$

and heat flux conditions

$$-(k\nabla T)\cdot \mathbf{n} = h(T - T_\infty) \; on \; \Gamma_q,$$

and $\Gamma = \Gamma_T \bigcup \Gamma_q$.

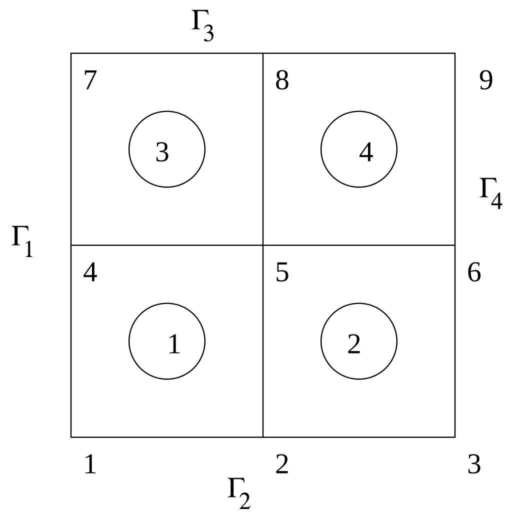
Show that the global equations have the form

$$[M]\{\dot{T}\} + [K]\{T\} + [H]\{T\} = \{Q\}.$$

Discretize the problem using the 4-node bilinear quadrilateral element and develop all the element-level operators (symbolically) in terms of the shape functions.

ME561-6

Using the results for the time-dependent heat equation from ME561-5, assemble the complete consistent capacitance matrix for nodes $1, 5, 8$ in the finite element mesh below. Assume that the grid has uniform mesh spacing $\Delta x = \Delta y = a$.



Develop the element-level row-sum lumped capacitance matrix in terms of the uniform grid spacing, a. Assemble the complete lumped mass matrix at nodes $1, 5, 8$. How do the entries of the row-sum lumped mass matrix correspond to the volume of each element? What approximations are being made with the lumped mass matrix?

ME561-7

Given a domain Ω with boundary Γ, develop the finite element model for

$$\rho C \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k_{xx}\frac{\partial T}{\partial x} + k_{xy}\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial y}\left(k_{yx}\frac{\partial T}{\partial x} + k_{yy}\frac{\partial T}{\partial y}\right)$$

with boundary conditions

$$T = \hat{T} \ on \ \Gamma_T,$$

and time-dependent heat flux conditions

$$-(k\nabla T)\cdot \mathbf{n} = sin(\omega t) \ on \ \Gamma_q,$$

and $\Gamma = \Gamma_T \bigcup \Gamma_q$.

Develop the global equations and formulate the generalized θ time-integrator for this problem. Using a 4-node bilinear quadrilateral element explain how the time-integrator treats the time-dependent heat flux boundary condition. Check the limits of $\theta = 0$ and $\theta = 1$ and explain how these choices affect the temporal accuracy.