Report 4 - Tooth Decay and the Wave Equation

A dentist that is tired of suffering the side-effects of low-dosage radiation from dental X-rays has proposed a "radiation-free" examination procedure to detect cavities in teeth. The device uses a probe that essentially consists of a wave guide that focuses a wave on the tooth and then measures the reflected signal to detect defects in the tooth. Figure 1 below illustrates the device.

As a technical consultant, you have been asked to provide a simulation capability to assess the performance of various oral probes and to assist in the design analysis cycle. There are currently two competing designs for the oral probe. One uses a solid stainless steel probe while the other uses a waveguide filled with air. The prototype oral probe has been designed with an overall length $L = 1.25\ cm$.

The input pulse to the probe consists of a truncated sinusoidal wave with a wavelength of 500 μm and peak displacement amplitude of 10 μm as shown in Figure 2. A "waveform transducer" is located 0.25 cm from the end of the oral probe and measures both the incident and reflected signal.

In order to assess each design, perform a series of demonstration computations where the input signal is propagated from the end of the oral probe to the tooth and back to the waveform transducer – approximately two transit times.

Complete the one-dimensional finite element code, "wave1d" to solve the wave equation,

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x} \left\{ E \frac{\partial u}{\partial x} \right\} + f, \tag{1}$$

where ρ is the mass density, and E is the elastic modulus (Young's modulus).

For the time integration algorithm, use the explicit central differences scheme ($\gamma = 1/2$, $\beta = 0$) with an "element-by-element" matrix-vector multiply. Include the capability to write the discrete solution, u^h, at intermediate time intervals for plotting purposes.

For the computations, use the following properties.

Property	Air	Stainless Steel
Density	$1\ kg/m^3$	$7600\ kg/m^3$
Elastic Modulus	$108,900\ Pa$	$190 \times 10^9\ Pa$
Sound Speed	$330\ m/s$	$5000\ m/s$

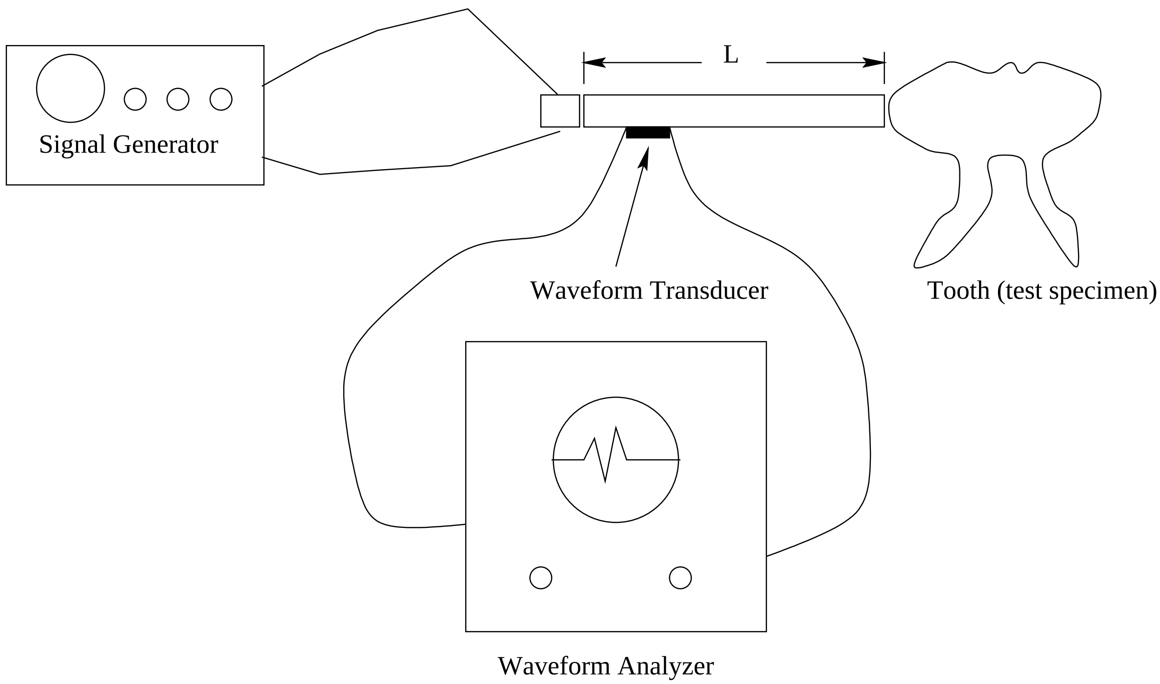

Figure 1: The megaherz pulsed hyperbolic tooth examination device.

For each case (air and stainless steel), run four CFL numbers ($CFL = 0.25,\ 0.5,\ 0.75,\ 1.0$) and plot the displacement field at the following non-dimensional times on the same axes. Here, $\tau = L/c$ is the transit time and c is the material sound speed. For all computations, use a grid with 250 elements and a uniform node spacing of $5 \times 10^{-3}\ cm$. Consider the "tooth" end of the probe to be rigidly attached to the tooth, i.e., no-displacement essential boundary condition.

$$\frac{t}{\tau} = \begin{cases} 1/4 \\ 1/2 \\ 3/4 \\ 1 \\ 5/4 \\ 3/2 \\ 7/4 \end{cases} \tag{2}$$

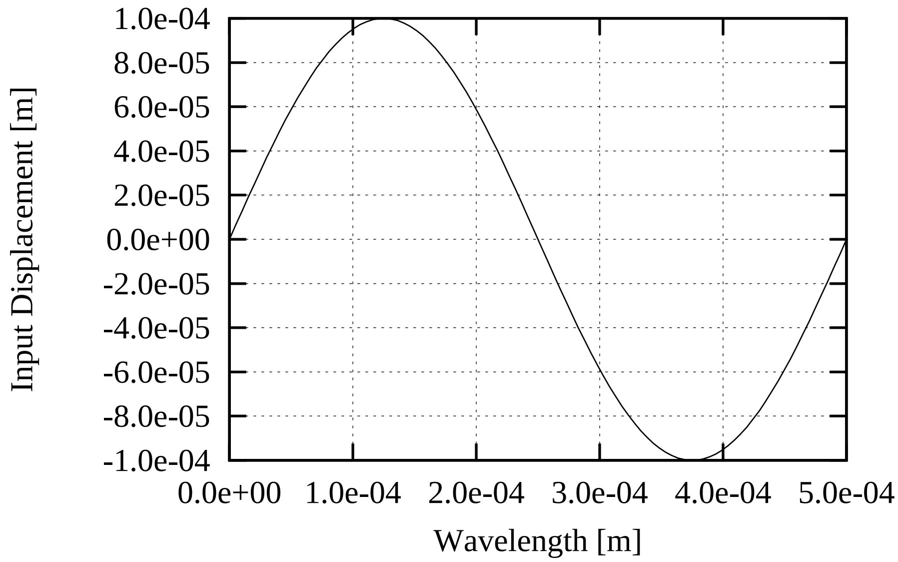

Figure 2: The driving sinusoidal waveform.