Report 2 - The Insulated Pipe

Develop a one-dimensional finite element code to solve the generalized ODE problem. Follow the guidelines for the code from the first report, i.e., the code must be capable of reading a control header, connectivity, coordinates and prescribed essential boundary conditions.

a) Perform a validation calculation that solves

$$\frac{d^2\theta}{dx^2} - 4x\frac{d\theta}{dx} + (4x^2 - 2)\theta = 0 \quad (1)$$

subject to the essential boundary conditions $\theta(0) = 1$, and $\theta(1) = 0$. Results should be compared to the exact solution

$$\theta(x) = exp[x^2] - xexp[x^2]. \quad (2)$$

b) It is necessary to insulate a water pipe that runs 2.12 km downhill from a warm spring to a cabin. The pipe is tapered so that the average velocity in the pipe decreases linearly from a maximum at the upper end to a smaller value at the lower end of the pipe. The maximum velocity that will be maintained at the smaller end is $U = 0.15\ m/s$. The equation that describes the variation of the temperature of the water in the pipe is

$$\frac{d}{ds}(k\frac{d\theta}{ds}) - \rho C_p U\frac{d\theta}{ds} + h(\theta_{\text{inf}} - \theta) = 0, \quad (3)$$

where

$\theta =$ Temperature $[^oC]$

$\theta_{\text{inf}} =$ Ambient air temperature (-30^oC)

$U =$ Mean water velocity $(0.15\ m/s)$

$\rho =$ Density $(1000\ kg/m^3)$

$C_p =$ Heat capacity $(4.211 \times 10^3\ N \cdot m/kg \cdot^o C)$

$k =$ Thermal conductivity $(0.574\ N \cdot m/s \cdot m \cdot^o C)$

$h =$ Convective heat transfer coefficient $(N \cdot m/m^3 \cdot s \cdot^o C)$

For the pipe, the amount of insulation is inversely proportional to h, therefore, it is desirable to determine the largest value of h for which the water in the pipe will not freeze. The large end of the pipe is to be placed at the spring where the fluid entering the pipe is at 20^oC. The water empties into a storage tank in the cabin where it is held at the temperature of the incoming water. This permits $d\theta/ds = 0$ to be used at the outlet of the pipe. Perform three computations to determine the largest convective coefficient h that may occur. Start with $h = 100$ as the initial guess.

Finally, perform a fourth computation with the outlet boundary condition being the specified temperature from your third trial analysis.

Plot the temperature profile as a function of s for all four cases and discuss the results.