# UDU/LDU Storage Format

This document outlines the storage format for the UDU and LDU direct solvers. As outlined in class, both the symmetric (UDU) and the non-symmetric (LDU) solver rely on an L-D-U decomposition of the coefficient matrix. Both the UDU and the LDU solver use a compact band-oriented storage scheme.

## UDU Storage Format

The UDU storage format stores only the upper-triangular portion of the global symmetric coefficient matrix in a band-oriented form. This is illustrated below.

$$\begin{bmatrix} & \textit{Original Matrix} & & & & \\ k_{11} & k_{12} & k_{13} & & & \\ k_{12} & k_{22} & k_{23} & k_{24} & & \\ k_{13} & k_{23} & k_{33} & k_{34} & k_{35} & \\ & k_{24} & k_{34} & k_{44} & k_{45} & k_{46} \\ & & k_{35} & k_{45} & k_{55} & k_{56} \\ & & & k_{46} & k_{56} & k_{66} \end{bmatrix} \Rightarrow \begin{bmatrix} \textit{UDU Format} & & \\ k_{11} & k_{12} & k_{13} \\ k_{22} & k_{23} & k_{24} \\ k_{33} & k_{34} & k_{35} \\ k_{44} & k_{45} & \\ k_{55} & k_{56} & \\ k_{66} & & \end{bmatrix} \quad (1)$$

The compact UDU storage scheme stores the diagonal in the first column of the array. The upper triangular part of the coefficient matrix is stored band-by-band in the remaining columns of the array. The total number of bands to be stored includes the diagonal and is specified by the 1/2-bandwidth, $N_{bw}$, of the symmetric matrix, i.e., the 1/2-bandwidth includes the diagonal.

## LDU Storage Format

In contrast to the UDU solver, the LDU solver requires that both the upper and lower triangular portions of the matrix be stored. Again, the matrix format for the LDU solver requires that coefficient matrix be stored band-by-band. This is shown below.

In the LDU compact storage scheme, the diagonal terms are stored in the "central" band with the lower-triangular terms stored in the "lower" bands and the upper-triangular terms stored in the "upper" bands. The compact

$$\left[\begin{array}{cccccc} & \multicolumn{5}{c}{Original\ Matrix} \\ k_{11} & k_{12} & k_{13} & & & \\ k_{21} & k_{22} & k_{23} & k_{24} & & \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & \\ & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} \\ & & k_{53} & k_{54} & k_{55} & k_{56} \\ & & & k_{64} & k_{65} & k_{66} \end{array}\right] \Rightarrow \left[\begin{array}{ccccc} \multicolumn{5}{c}{LDU\ Format} \\ & & k_{11} & k_{12} & k_{13} \\ & k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} \\ k_{42} & k_{43} & k_{44} & k_{45} & k_{46} \\ k_{53} & k_{54} & k_{55} & k_{56} & \\ k_{63} & k_{64} & k_{66} & & \end{array}\right] \quad (2)$$

storage scheme requires $N_{bw}$ bands where the bandwidth in this case is the total bandwidth of the coefficient matrix. The diagonal band is stored as the $N_{bw}/2+1$. In this simple example, the bandwidth is 5 and the diagonal terms are stored in the third band.